The Rise of Private Money as a Competing Medium of Exchange*

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Abstract  
When it is costly for a store (i.e., clearinghouse) to mediate trade, a speculative demand for money may arise. Trading money to another agent for a good, rather than trading good for good through a store, allows the money holder to obtain the good while avoiding the fee the store must charge to cover its costs. We show that when the mediation market is contestable, there are conditions in which the store must create and issue a form of fiat money to maintain control of the market. The equilibrium quantity of money is private money, determined by agent expectations and market forces.

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1. Introduction

If a clearinghouse can effectively mediate trade, is there still a role left for money? Money would not be needed to resolve the double coincidence of wants problem. However, we show that if it is costly for a “store” to provide mediation, there is still a role for money because holding it provides traders the opportunity to avoid the fee the store must charge to cover its costs. We also show that, when the store’s mediation technology is freely available so the mediation market is contestable, the store will be motivated to create the fiat money, and the quantity of fiat money circulating in equilibrium depends upon trading environment parameters. That is, we present a model of private money, where the quantity supplied is endogenously determined.

This complements other recent work seeking to explain how money may arise and circulate. Starr (2003), for example, shows how an Arrow-Debreu general equilibrium model can be modified so money arises endogenously. The key modifications are a sequential trading restriction and transactions costs. Under certain conditions, all agents choose to trade for any good by first trading through the good or intrinsically useless object that elicits the lowest transactions cost, or the money. If marginal transactions costs decrease as the volume of trade increases, money can arise even when there is no double coincidence of wants problem. This leads Starr (2003, p. 456) to suggest “the focus on the absence of double coincidence of wants – as distinct from transaction costs – as an explanation for the monetization of trade may miss a significant part of the underlying causal mechanism.”

1 We use the term store for our mediator, in part to avoid confusing our mediation conception with other conceptions in the literature, including trading post and middleman. Our store is closest in form to the clearing house associated with the Walrasian auctioneer in general equilibrium models.

2 Starr’s model captures Tobin’s (1980) suggestion that “The use of a particular language or a particular money by one individual increases its value to other actual or potential users.” Starr further uses his model to explain the circulation of fiat money by imposing the assumption that only the government sponsored money can be used to pay taxes. When trading volume reduces the average transactions cost, the volume of trade necessitated by the payment
different in form. Our store eliminates the double coincidence of wants problem, yet money can still arise because of a transactions cost, specifically the store’s operation cost.

Because our model is an extension of the KW (Kiyotaki and Wright, 1993) search theoretic model, our work is less related to Starr (2003) and more related to Ritter (1995), Williamson (1999), Cavalcanti et al (1999), and Berentsen (2006). Ritter (1995, p. 140) claims the KW model is incomplete because it does not explain where money comes from. He shows that if a coalition of KW agents is large enough, it will have the credibility and incentive to successfully issue fiat money. Using a variant of Ritter’s model, Berentsen (2006) demonstrates the relevance of public knowledge and the ability to punish the money issuer, for supporting the circulation of money when it is being issued by a private, revenue-maximizing monopolist. Williamson (1999) develops a model where banks issue bank notes to mitigate a mismatch between the receipt of investment payoffs and the desired timing of consumption. Cavalcanti, et al. (1999) create a structure where banks are motivated to issue bank notes to obtain profit from float; the bank can consume when the note is issued, but does not have to pay for the consumption until the note is redeemed. None of these innovative explanations for the
introduction of money are based upon the idea that money exists and circulates because it effectively competes with another medium of exchange, which is the idea behind our model.

Shi (2006) identifies three ways researchers have introduced mediums of exchange that can compete with money in the KW framework: mechanism design, bilateral credit, and middlemen, and our store which we describe below is tangentially related to each. Mechanism design involves creating a resource allocation mechanism that is a form of public record keeping and compatible with the incentives of the agents. Using this approach, Kocherlakota (1998) shows money can play an essential role only if the public record keeping device is imperfect. Bilateral credit involves an agent being able to issue a nontransferable IOU to complete a trade when there is only a single coincidence of wants. Credit is an imperfect substitute for money because the creditor must not consume and monitor the debtor until the debtor is in a position to repay the IOU. This allows bilateral credit and money coexist in equilibrium, and Shi (1996) shows the introduction of bilateral credit enhances efficiency. Adding middlemen has involved either assuming middlemen can improve information about quality of the good being purchased by making an investment, or assuming middlemen improve the likelihood of the match by investing to acquire capacity to store more types of goods. With either of these approaches, money and trade through middlemen can coexist when the investment cost for middlemen is not too high.

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6 Howitt (2005) uses a creative modification of the Starr (2003) framework, where profit seeking firms create shops that reduce agent search costs by easily located. This approach can be viewed as providing a microfoundation for the trading post story of the type introduced by Shapley and Shubik (1977). Because Howitt’s shops have fixed costs of operation and can only mediate the trade of a limited number of goods, money can arise as an additional medium of exchange, for firms must accept money in exchange for goods in order to generate enough trading volume to cover the fixed costs.

7 This discussion of bilateral credit is based upon Shi (1996). See Li (2001) and Corbae and Ritter (2002) for other examples.

8 See Li (1999).

9 See Johri and Leach (2002) and Schevchenko (2004).
The microfoundation for our model is that of KW (1993), as restated by Ljungqvist and Sargent (2000, pp. 602-604). Without a clearing house or money, the usual assumption that agents specialize in consumption and production creates a trading friction. Agents can barter in this decentralized market bilaterally if they are randomly matched and experience a double coincidence of wants. Following the approach initiated by Lagos and Wright (2005), we also allow agents to participate in a centralized market, though our construction is slightly different. Refraining from modeling the alternative to money in a decentralized manner, in some manner similar to one of those described in the previous paragraph, allows us to keep the model simple and focus on the competition for transactions that can develop between the store and money. Our centralized market is a stylistic clearing house we call a store. We assume our store can purchase a technology that allows it to mediate trade like a Walrasian auctioneer by paying a transactions cost. As this store operation cost goes to zero, there is no scope for money to arise or circulate because the store’s mediation is entirely efficient. However, when the operation cost is positive, the store must charge a fee on transactions to cover its costs, and it is this that allows money to compete. Thus, one can say that it is a speculative demand for money that motivates agents to hold it as a store of value and use it a medium of exchange.

But, how does money arise? We show a contestability assumption is sufficient for providing the store with the motivation to innovate and introduce a form of money. When the mediation market is contestable, the store must not only charge efficient mediation fees to avoid being displaced, it must also avoid being displaced by a more innovative store. Our standard store provides agents the opportunity to trade good for good. The monetary innovation occurs when this standard store also offers agents the opportunity to trade good for a transferable IOU, like today’s store value cards or gift cards. When an agent delivers good to the store in exchange for the IOU, consumption is foregone. However, holding the IOU gives the agent the opportunity to trade it to another agent for good in the decentralized market and avoid paying the store’s markup. The IOU becomes circulating money when agents are willing to

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10 He, Huang, and Wright (2005, p. 639) similarly contend that introducing a centralized market is a useful simplification, one that allows them to more easily develop and examine banking sector in the KW framework.
accept it in exchange for good and when the store must issue it in order to prevent entry. Our existence result involves identifying the environmental conditions that will support the creation and circulation of money in this manner.\footnote{This is monetary theory with a microfoundation, as defined by Shi’s (2006, p. 646): That which “derives the role of money from the trading environment.”}

Money will not arise nor circulate in our economy unless agents have sufficient willingness to accept it in exchange for good. As in KW (1993), there is a non-monetary equilibrium, which occurs when there is insufficient confidence in money. However, unlike KW (1993) confidence need not be complete in our model for an equilibrium to exist. There is a continuum of rational expectation monetary equilibria, parameterized by the willingness of agents to accept money. Above the threshold, there is a unique equilibrium for each value of the money confidence parameter, and the equilibrium quantity of money increases as the confidence in money increases.

The money that circulates in our model may be considered fiat money, even though it is backed by the store’s promise to redeem it for good. Shi (2006, p. 644), referring to Wallace (1980), defines fiat money as a circulating object that is (a) intrinsically useless and (b) not backed by government policy. The money in our model is fiat money by this definition. Moreover, it is incentive compatible for an agent in our model to hold money only if the agent never expects to return the money to the store. The store’s offer to redeem the money is not non-credible cheap talk, but it inconsequential talk. The willingness to accept money in exchange for good is supported by the confidence agents have in money being accepted by other agents in future periods, not by the willingness of the store to redeem the money.

Any monetary equilibrium for our economy is stable. When the quantity of money is above the equilibrium value, agents who do not barter prefer trading good to the store for good, not for money, and money holders also prefer trading money to the store for good, rather than holding it. Therefore, money holding decreases. This reduction in money holding makes it more likely that holding money will produce a match where money can be traded for good, making holding money more attractive. The converse analysis applies when the quantity of money is below the equilibrium level. Consequently,
The equilibrium quantity of money in our economy is almost surely not welfare maximizing, and money may or may not be essential. This is because the decision to hold money generates externalities which agents do not internalize: It reduces the economy’s output level and makes trade between agents less likely. When barter is easy enough or agents don’t discount the future enough, more money only reduces welfare. Alternatively, when barter is difficult enough or agents discount the future enough, the effect of money on welfare depends upon the operating cost level of the store. At a low store operating cost, more money only increases welfare. At a high store operating cost, more money only decreases welfare. At intermediate store cost levels, there is a welfare maximizing quantity of money. This optimal quantity of money would arise if and only if the willingness of agents to accept money happens to be just right. For money to be essential, it is necessary that the store have a relatively low operating cost, and either barter must be relatively difficult or agents must discount the future at a relatively high rate.

The organization of the paper is as follows. Section 2 presents our economy’s microfoundation, which we label the Barter Economy. Section 3 reviews how the introduction of fiat money into this Barter Economy by government can facilitate trade and enhance welfare, if agents are sufficiently willing to trade for and hold fiat money. In section 4, we return to the Barter Economy and introduce a store that we assume can mediate trade when barter fails, which provides the clearing house trading environment we need for section 5. In section 5, we develop a model where the existing store in a contestable mediation market recognizes the need to issue a form of fiat money in order to maintain control of the mediation market. We define an equilibrium for this money-store economy, provide an existence

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12 Shi (2006, p. 644) defines money as being “essential” if it “improves the efficiency of resource allocations relative to an economy without money.” He argues essentiality is a property to be sought in a model of money because (a) we want to understand how much money can improve welfare, and (b) a model with non-essential money is not likely representative because it would not likely stand the test of time.
theorem, and a stability theorem. In section 6, we examine welfare issues, and we conclude with some discussion in section 7.

2. The Barter Economy

The Barter Economy presented here is a discrete time version of the Kiyotaki and Wright’s (1993) model, very closely related to that described by Ljungqvist and Sargent (2000, pp. 602-604). There is a continuum of infinitely lived agents and a continuum of goods, each normalized to one. Agents are specialized in consumption. The exogenous variable \( x \) denotes the proportion of goods that can provide utility to a given agent, and \( x \) also equals the proportion of agents that can obtain utility from consuming any particular good. The utility \( u \) of consuming \( y \) units of a consumable good is given by \( u(y) = y \cdot 13 \)

Agents are also specialized in production. Production is a random draw from the continuum of goods, which yields one unit of a particular good to the producing agent. The produced good does not provide utility to the agent, but can be stored without cost. All agents initially produce, so all begin the first period with good. Production may only occur at the moment when one time period proceeds to the next.

In each period, there is a social interaction modeled as a bilateral random matching process. The probability that one agent is matched with another is \( \theta \), where

\[
0 < \theta \leq 1.
\]

Because no good has any special characteristics, the probability that a good will be accepted in exchange is independent of the good held by an agent. Thus, \( x \) is the probability that any given trader will want any given good in exchange (i.e., the probability of a single coincidence of wants), and \( x^2 \) is the probability an agent in a match will experience a double coincidence of wants. It is assumed

\[
0 < x < 1,
\]

so a double coincidence of wants may or may not occur when two agents are matched.

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13 This utility function is beyond what is needed here, where there is no technology available for dividing goods, but it will useful once a store technology is introduced that allows goods to be divided.
Each agent experiences the disutility $\varepsilon$ whenever a good is accepted in trade, and

$$0 < \varepsilon < 1.$$  \hspace{1cm} (3)

This positive transactions cost rules out the formation of commodity money. Trading for a non-consumable good would generate the cost $\varepsilon$ without positively affecting future trading or consumption opportunities. Thus, an agent will only trade for a consumable good. Further, an agent will trade and consume if a double coincidence of wants occurs because the agent is allowed produce after consumption and enter the next period with a unit of good. Because no agent can divide a good, each agent trades one unit of good for one unit of good in a barter transaction and experiences the net utility $u - \varepsilon = 1 - \varepsilon$.

Each agent discounts the future at the same constant rate. The discount factor $\beta$ denotes the current period value of one unit of utility received in the next period. Agents discount the future to some extent, but not entirely, implying

$$0 < \beta < 1.$$  \hspace{1cm} (4)

Looking forward from any time period, the value function for an agent in the Barter Economy is

$$V^B_G = \theta \varepsilon^2 (1-\varepsilon) + \beta V^B_G,$$

which implies each agent expects a lifetime discounted expected utility of

$$V^B_G = \frac{\theta \varepsilon^2 (1-\varepsilon)}{1-\beta}.  \hspace{1cm} (5)$$

3. The Kirotaki-Wright Fiat Money Economy

Kirotaki and Wright (1993) show that, if agents have enough confidence that fiat money will be accepted in exchange for good, then the circulation of fiat money can be supported, transforming the Barter Economy into a Fiat Money Economy and improving welfare when barter is difficult enough. Following the presentation of Ljungqvist and Sargent (2000), we present a Fiat Money Economy here that we can use for comparison purposes below.

Fiat money is introduced to the Barter Economy from an outside source; e.g. government. A fraction $M \in [0,1]$ of agents are offered one unit of a durable item with no intrinsic value. Agents who accept the fiat money cannot produce. This implies agents within a given period can be divided into two
types: Good holders and money holders. No agent is forced to hold money, but rather must choose to hold it. For a Fiat Money Economy to arise, a fraction of agents must choose to hold money.

Let $V_G^M$ and $V_M^M$ denote the values associated with beginning a period with good and money, respectively. The Bellman equations for these two states can be written

\[
V_G^M = \theta \left[ 1 - M \right] x^2 \left[ 1 - \varepsilon + \beta V_G^M \right] + \theta M x \max_p \left[ p \beta V_M^M + (1 - p) \beta V_G^M \right] + \left[ 1 - \theta (1 - M) x^2 - \theta M x \right] \beta V_G^M
\]

\[
V_M^M = \theta \left[ 1 - M \right] x P \left[ 1 - \varepsilon + \beta V_G^M \right] + \left[ 1 - \theta (1 - M) x P \right] \beta V_M^M.
\]

A good holder barters, or exchanges good for money, or holds over good to the next period, which is why there are three terms on the right side of equation (6). The first term is associated with barter, which occurs with probability $\theta (1 - M) x^2$, when the good holder is matched with another good holder and a double coincidence of wants occurs. In this event, $1 - \varepsilon$ units of utility are obtained from current consumption and $\beta V_G^M$ units of utility are obtained from all future transactions, associated with producing and starting the next period as a good holder. The second term is associated with the possibility of trading good for money, which occurs with probability $\theta M x$ when the good holder is matched with a money holder and the money holder wants the good held by the good holder. The good holder chooses the probability $p \in [0,1]$ of accepting money that maximizes the expected future utility of holding money, recognizing that accepting money will yield $\beta V_M^M$ while holding over good will yield $\beta V_G^M$. If no barter and no trade for money occur, then the good holder holds the good over to the next period, which yields $\beta V_G^M$ with probability $1 - \theta (1 - M) x^2 - \theta M x$.

A money holder trades the money for good or holds the money over to the next period, which is why two terms appear on the right side of equation (7). To be able to trade money for good, the money holder must be matched with a good holder and the good holder must be holding a consumable good. This occurs with probability $\theta (1 - M) x$. However, the good holder may not accept the money. Let $P \in [0,1]$ denote the probability that a randomly chosen good holder will accept money in exchange for
good, and assume this is the money holder’s belief regarding the probability that money will be accepted. It follows that the money holder believes money will be traded for good with probability $\theta(1-M)xP$. This event provides $1-\epsilon$ units of utility in current period consumption and $\beta V^M_G$ units of utility from future transactions. The alternative no trade event occurs with probability $1-\theta(1-M)xP$, which yields $\beta V^M_M$ units of utility, associated with starting the next period as a money holder.

An equilibrium for the Fiat Money Economy is a state $(P,M)$ such that $P = p$, so expectations are rational, and $M \in [0,M]$, so the fraction of money holders is consistent with the fraction of agents initially provided the option to hold money. Following Kirotaki and Wright (1993), Ljungqvist and Sargent (2000) show that there are three values for the belief $P$ consistent with the rational expectation $P = p$. These are $P = 0$, $P = 1$, and $P = x$. The corresponding fractions of agents holding money in equilibrium are $M = 0$, $M = M$, and $M \in [0,M]$. These results follow from the fact that $P < x$ implies $V^M_M < V^M_G$, and agents maximize by choosing $p = 0$ (never accept money), while $P > x$ implies $V^M_M > V^M_G$ and agents maximize by choosing $p = 1$ (always accept money). Thus, we learn that the circulation of fiat money can be supported if people have enough faith in it.

The introduction of money as a medium of exchange can improve welfare. Using the welfare criterion $W = MV^M_M + (1-M)V^M_G$ to examine well being in the pure monetary equilibrium (where $P = 1$), Ljungqvist and Sargent (2000) show that the introduction of fiat money can increase welfare as long as $x < 1/2$. That is, money enhances well-being, or is “essential,” if barter is difficult enough. Assuming $x < 1/2$, the quantity of money that maximizes societal welfare is $M^* = 1 - 2x/(2-2x)$, and maximum societal welfare is $W^* = \left[\theta(1-\epsilon)x(1-x)\right]/\left[(1-\beta)(2-2x)^2\right]$.

4. The Store Economy

Here, rather than introducing money, we introduce a highly stylized store into the Barter Economy. A store mediates trade by accepting the one unit of good from an agent, providing the
amount $\gamma$ of some other good in exchange. The store’s technology, which the store must purchase at a positive cost, allows the store to divide goods so $\gamma$ need not equal one. The store must markup the goods it buys to cover its costs, and the store seeks profit. The fee $1 - \gamma$ an agent must pay for trading through the store is the same for each good and is public information, so agents make their plans observing $\gamma$ in the range

$0 \leq \gamma < 1$. 

The positive markup implies trading through the store is a second best option for agents, adopted if and only if the social interaction does not yield a barter transaction. We assume the store’s technology allows the store to fully coordinate the trade of agents who do not barter. A rationale for this assumption is the following. When the $1 - \theta$ agents are not matched, it is not because they cannot be matched, but because they do not happen to meet another agent. An advantage of a store is people know its location. If the $1 - \theta$ agents not matched in the social interaction go to the store, the store can ensure each is matched with another. The store would also have the opportunity to coordinate trade for the $\theta [1 - x^2]$ agents who are matched with another in the social interaction but do not experience a double coincidence of wants. Thus, there will be $1 - \theta x^2$ agents who arrive at the store because they do not complete a barter transaction. Because production and matching each occur at random, the preferences of these $1 - \theta x^2$ agents would be randomly distributed, just as all agents preferences were distributed before the social interaction. If these agents could have experienced a double coincidence of wants in the social interaction, but did not because the matching process is random rather than selective, then it is reasonable to think that the store could, by paying a cost to implement a selective matching process, coordinate the trade of any measure of agents.

Assuming trade through the store also generates the transactions cost $\varepsilon$ for agents, no agent would be willing to trade through the store if $\gamma < \varepsilon$. Trading through the store in this case would
produce a net current period loss of utility, while holding over good to the next period would not. Thus, the store must set the trading rate so

\[ \gamma \geq \varepsilon. \]  

The value function for an agent in the Store Economy is

\[ V^S_G = \beta x^2 \left[ 1 - \varepsilon + \beta V^S_G \right] + \left[ 1 - \beta x^2 \right] \gamma - \varepsilon + \beta V^S_G. \]  

The first term on the right side of (10) is the value associated with the barter possibility, while the second term is the value associated with the opportunity to trade through the store. Solving for \( V^S_G \), each agent expects the lifetime discounted expected utility

\[ V^S_G = \beta x^2 \left[ 1 - \varepsilon \right] + \left[ 1 - \beta x^2 \right] \gamma - \varepsilon \]

Comparing (11) and (5), we see that the introduction of the store increases the expected lifetime utility of the agent if and only if \( \gamma > \varepsilon \).

The store captures as revenue the fraction \( 1 - \gamma \) of each unit of good delivered by an agent, and \( 1 - \beta x^2 \) agents trade through the store. Therefore, measured in units of utility, the expected total revenue of the store is \( \left[ 1 - \gamma \right] \left[ 1 - \beta x^2 \right] \).

The store operation cost is measured in units of utility, and can be paid by the store using any good. Because the form of the cost will matter in the monetary economy below, we consider two cases. In one case, the store’s operating cost is the fixed cost

\[ \bar{c} > 0. \]

In the other case, the store’s operating cost is the variable per unit cost

\[ \hat{c} > 0. \]

Using these cost assumptions, the store’s profit level is \( \pi = \left[ 1 - \gamma \right] \left[ 1 - x^2 \right] - \bar{c} \) in the fixed cost case and \( \pi = \left[ 1 - \gamma - \hat{c} \right] \left[ 1 - x^2 \right] \) in the variable cost case. A store is viable if and only if it earns a non-negative profit; i.e., \( \pi \geq 0 \).
The store is a profit maximizing entity. If a profit is earned it would be in the form of good that could be distributed back to the owners of the firm, which would enhance the well being of the agent owners. It is convenient to examine a contestable market equilibrium. When the mediation market is contestable, the existing store must prevent other stores from capturing the market. It turns out that, to do this, the existing store must act so as to earn zero profit. If the existing store does not earn a profit, then there is no need to model how profits are distributed, which simplifies the model. Given conditions (1)-(4) and (12)-(13), a Contestable Market Equilibrium for the Store Economy is a value for $\gamma$ such that (i) the store is viable; i.e., $\pi \geq 0$, (ii) the store attracts customers; i.e., $\gamma \geq \varepsilon$, (iii) no other store can enter and capture the mediation market from the existing store; i.e. there is no $\gamma' > \gamma$ yielding store profit $\pi' \geq 0$, and (iv) the store’s profit $\pi$ is maximized under conditions (i)-(iii).

Theorem 1 (Existence of a Contestable Equilibrium for the Store Economy): A Contestable Market Equilibrium for the Store Economy exists if and only if

$$\bar{\varepsilon} \leq \left[1 - \delta \varepsilon^2 \right] \left[1 - \varepsilon \right]$$  \hspace{1cm} \text{(fixed cost case)}  

(16)  

$$\hat{\varepsilon} \leq 1 - \varepsilon$$  \hspace{1cm} \text{(variable cost case)}  

(17)  

Proof: See the Appendix

In either the fixed or variable cost case, the store’s desire to maximize its profit is superseded by its need to retain control of the contestable market. The trading rate $\gamma$ in the contestable market equilibrium is as attractive as it can be for agents because it must be set so profit equals zero to prevent entry. In the fixed cost case, the equilibrium trading rate is $\gamma = \left[1 - \delta \varepsilon^2 \right] / \left[1 - \varepsilon \right] = 1 - \bar{\varepsilon} / \left[1 - \delta \varepsilon^2 \right]$. This rate worsens as the double coincidence likelihood increases because the store has fewer customers over which to spread the fixed cost. Using this trading rate to compare life in the Store Economy to that in the Barter Economy, we find $V^S_G - V^B_G = \left[1 - \delta \varepsilon^2 \right] / \left[1 - \beta \right] - \bar{\varepsilon} / \left[1 - \beta \right]$, which implies a store will improve well-being when it can arise. In the variable cost case, the equilibrium trading rate $\gamma = 1 - \hat{\varepsilon}$ is
independent of the volume of trade through store. Using this rate, we find

\[ V^S_G - V^B_G = \left[ 1 - \varepsilon \right] - \left[ 1 - \theta \varepsilon^2 \right] \hat{c} / \left[ 1 - \beta \right]. \]

Again, for the variable cost case, we find the store will enhances well-being if it can arise.\(^{14}\)

In both the fixed cost and variable cost cases, as the cost of store operation goes to zero, the equilibrium trading rate \( \gamma \) goes to one. With zero store operation costs, the store in a contestable market provides a perfectly efficient medium of exchange, and the expected lifetime utility of an agent is

\[ V^S_G = \left[ 1 - \varepsilon \right] / \left[ 1 - \beta \right]. \]

5. A Money-Store Economy

The existence theorems for the Store Economy indicate there is reason to think a store would naturally arise in the Barter Economy. Individual agents have an incentive to introduce it. This begs another question. If the store solves the double coincidence of wants problem in the Barter Economy, then is there any niche left for money? We will now show there is a remaining niche. By recognizing this niche, we explain why money might naturally arise as a second medium of exchange in the economy. The remaining niche is a speculative niche, for holding money as a medium of exchange potentially allows the money holder to avoid the store’s markup.

Suppose, in addition to being able to trade good for good, an agent can also choose to accept an IOU from the store valued at \( \gamma \) units of desired good. Because the agent discounts future consumption, there is no reason to hold the IOU unless the store offers some additional incentive. The incentive we explore is a store policy that allows the IOU to be traded, so the agent who redeems the IOU at the store need not be the agent who initially receives the IOU from the store. When the IOU is held from one period to the next, it becomes money held as a store of value. When the IOU is traded from one agent to another, it becomes money circulating as a medium of exchange.

\(^{14}\) Store viability implies \( \hat{c} \leq 1 - \varepsilon \) and we know \( 0 < 1 - \theta \varepsilon^2 < 1 \). Together, these two conditions imply

\[ 1 - \varepsilon > \left[ 1 - \theta \varepsilon^2 \right] \hat{c}, \]

so

\[ V^S_G - V^B_G = \left[ 1 - \varepsilon \right] - \left[ 1 - \theta \varepsilon^2 \right] \hat{c} / \left[ 1 - \beta \right] \]

implies \( V^S_G > V^B_G \).
As is typical, assume production cannot occur as long as money is held. Let $M$ denote the measure of agents holding money, so $1 - M$ is the measure not holding money. Figure 1 presents the trading possibilities for an agent in the Money-Store Economy, depending upon whether the agent enters the given period holding money or holding good.

**Figure 1: The Money-Store Economy**

Let $V_M$ and $V_G$ denote the lifetime discounted expected utility of an agent who starts a period holding money and good, respectively. The value function for a money holder can be written as

\[
V_M = \theta [1 - M] x P [1 - \varepsilon + \beta V_G] + \left[1 - \theta [1 - M] x P \right] \max \left\{ p \beta V_M + \left[1 - p \right] y - \varepsilon + \beta V_G \right\}.
\]

The first term is associated with the possibility of trading money for good. The probability is $\theta [1 - M]$ that an agent holding money is matched with an agent holding good, and $x$ is the probability that the agent holding money wants the good held by the other. Thus, the probability of a “money match” is
\( \theta[1-M]x \). When a money match occurs, the money holder moves to a chance node, where the agent holding good decides whether or not to accept the money. We assume all agents, including the money holder, subjectively perceive other agents will accept money with probability \( P \in [0,1] \). If the money is accepted, the money holder trades money for good, consumes, produces, and enters the next time period holding good. This transaction occurs with probability \( \theta[1-M]xP \) and yields \( 1 - \varepsilon + \beta V_G \) units of lifetime expected utility, explaining why \( \theta[1-M]xP[1 - \varepsilon + \beta V_G] \) appears in the value function.

The second term of value function (18) is associated with a money holder who is not able to trade money to another agent for good. This disappointing outcome occurs with probability \( 1 - \theta[1-M]xP \), and the agent holding money proceeds to a choice node, where a decision must be made as to whether to continue holding money or not. This choice involves solving the problem

\[
\max_{p \in [0,1]} \{ p \beta V_M + [1 - p] \gamma - \varepsilon + \beta V_G ] \},
\]

where \( p \) is the probability with which the agent chooses to continue to hold money. When the agent continues to hold money, consumption is forgone, no production occurs, and the next period is experienced as a money holder, which yields an expected lifetime utility of \( \beta V_M \). Alternatively, when the agent decides against holding money, \( \gamma \) units of good are received from the store, consumption occurs, production occurs, and the next period is experienced as a good holder, which yields the expected lifetime utility of \( \gamma - \varepsilon + \beta V_G \). When \( \gamma - \varepsilon + \beta V_G < \beta V_M \), the pure money holding strategy \( p = 1 \) is best. When \( \gamma - \varepsilon + \beta V_G > \beta V_M \), the pure strategy of not holding money \( p = 0 \) is best. When \( \gamma - \varepsilon + \beta V_G = \beta V_M \), the agent is indifferent between holding money and not, so any mixed strategy \( p \in [0,1] \) is as good as any other such strategy. Regardless of the solution of problem (19), the
second term of the value function, \([1 - \theta[1 - M]x_P]Max_p\{p\beta V_M, [1 - p]y - \varepsilon + \beta V_G]\), represents value an agent holding money expects when the agent is unable to trade money to another agent for good. \(^{15}\)

The value function for a good holder can be written as

\[V_G = \theta[1 - M]x^2[1 - \varepsilon + \beta V_G] + [1 - \theta[1 - M]x^2]Max_p\{Max_p\{p\beta V_M + [1 - p]y - \varepsilon + \beta V_G]\}, \beta V_G\}\]

The first term of this value function comes from the barter possibility. The probability of being matched with another good holder is \(\theta[1 - M]\), and the probability of double coincidence of wants is \(x^2\). Therefore, the probability that a good holder will experience a barter trade is \(\theta[1 - M]x^2\). When barter occurs, each agent in the pair consumes, produces, and enters the next period starting with good. Thus, barter yields \(1 - \varepsilon + \beta V_G\) units of utility. Because this is at least as much utility as any other possibility, a barter trade is executed whenever possible. Consequently, the expected value associated with the barter possibility is \(\theta[1 - M]x^2[1 - \varepsilon + \beta V_G]\).

The second term of value function (20) is associated with the possibility that the agent does not complete a barter transaction, which occurs with probability \(1 - \theta[1 - M]x^2\). In this event, the agent must decide whether or not to hold over the good to the next period. Utility \(\beta V_G\) is obtained from holding over the good, and \(\text{Max}_p\{p\beta V_M + [1 - p]y - \varepsilon + \beta V_G\}\) is obtained from solving problem (19) otherwise. To understand why problem (19) is always faced when good is not held over, first note that one reason for not completing a barter trade is the agent is matched with another good holder, but no double coincidence of wants occurs. In this case, the agent holding good proceeds to the store and solves problem (19) to determine whether to accept money from the store or trade good for good. Alternatively,

\(^{15}\) In order for an agent to have ever decided to hold money \(\beta V_M \geq y - \varepsilon + \beta V_G\) would have to have held at some point in the past. Thus, in order to have this problem to solve, we know that the agent will not choose \(p = 0\), and we could present the value function of the money holder as \(V_M = \theta[1 - M]x_P[1 - \varepsilon + \beta V_G] + [1 - \theta[1 - M]x_P]\beta V_M\). We choose to present the more general value function for the money holder as we do in (18) so that the reader can see all of the possible trading options in the value function, (even those that will not be used in equilibrium).
barter might not occur because the good holder is matched with a money holder. If the money holder
does not want the good held by the good holder, then the good holder again proceeds to the store and
solves problem (19). If the money holder wants the good holder’s good, the good holder must decide
whether to accept the money holder’s money, or not. This again involves solving problem (19) for the
alternative is trading the good to the store in exchange for good, as long as holding over good is not
better.

At this point, it is useful to illustrate why the introduction of money offers potential benefit. The
probability is \( p \theta M x \) that a good holder will trade away good for money, for this requires that the good
holder be matched with a money holder, that the money holder wants the good holder’s good, and that the
good holder will accept the money. Because a good holder experiences barter with probability
\( \theta [1 - M] x^2 \), the probability that an agent holding good neither experiences a barter match nor trades after
a money match is \( 1 - \theta [1 - M] x^2 - p \theta M x \). In this case, the agent proceeds to the store. In the Store
Economy, the probability that a good holder will arrive at the store is \( 1 - \theta x^2 \). Subtracting the former
probability from the latter, we find the difference is \( \theta M x [p - x] \), which is positive and increasing in \( M \) when \( p > x \). Thus, when the likelihood a good holder will accept money exceeds the likelihood of a
single coincidence of wants, more money in circulation makes it less likely that good holders will have to
go to the store and pay the store’s markup. This is why the circulation of money has the potential to
increase well being.

We now turn to the store’s problem. For the store to exist, condition (9) must hold, and if \( \gamma \geq \varepsilon \)
then no good holder will ever hold over good from one period to the next, for nothing is gained. This
implies all agents who arrive at the store holding good will either trade good for good or trade good for
money. Let \( \alpha \) denote the fraction that trade good for good, so \( 1 - \alpha \) trade good for money. Let \( \delta \)
denote the fraction of those arriving at the store with money that trade money for good, so that
\( 1 - \delta \) choose to hold money. It follows that the store experiences the following transactions. There are
agents who deliver good to the store in exchange for good. There are
\[ [1 - \alpha] (1 - \theta[1 - M] x^2 - p \theta M x) \] agents who deliver good to the store in exchange for money. There are
\[ \partial M [1 - \theta[1 - M] x P] \] agents who deliver money to the store in exchange for good. It follows that the
revenue of the store is
\[ (21) \ R = [1 - \gamma] \alpha [1 - M] (1 - \theta[1 - M] x^2 - p \theta M x) + [1 - \alpha] (1 - \theta[1 - M] x^2 - p \theta M x) \gamma \partial M [1 - \theta[1 - M] x P]. \]

To keep the model as simple as possible, we assume the store sets its trading rate \( \gamma \) with the
expectation that a stationary equilibrium will prevail, where the new money it issues will equal to old
money it redeems, so
\[ (22) \ [1 - \alpha] (1 - \theta[1 - M] x^2 - p \theta M x) = \partial M [1 - \theta[1 - M] x P]. \]

Condition (22) and revenue definition (21) together imply the revenue function for the store reduces to
\[ (23) \ R = [1 - \gamma] [1 - M] (1 - \theta[1 - M] x^2 - p \theta M x). \]

Examining condition (23), we see that, as in the store economy, the store’s revenue depends upon
the markup \( 1 - \gamma \) and the number of agents holding good who end up trading through the store, which is
\[ [1 - M] (1 - \theta[1 - M] x^2 - p \theta M x). \] The store’s equilibrium profit level is
\[ \pi = [1 - \gamma] [1 - M] (1 - \theta[1 - M] x^2 - p \theta M x) \bar{\pi} \text{ for the fixed cost case and } \]
\[ \pi = [1 - \gamma - \bar{\epsilon}] [1 - M] (1 - \theta[1 - M] x^2 - p \theta M x) \] for the variable cost case. We can now define an
equilibrium for the Money-Store Economy:

**Definition (Equilibrium for Money-Store Economy):** Given conditions (1)-(4) and (12)-(13), an
equilibrium for the Money-Store Economy is a quadruple \( (p, P, \gamma, M) \) such that (i) agents maximize
their lifetime expected utility; i.e., the value functions (18) and (20) are satisfied, (ii) each agent holds
money if and only if it is optimal; i.e., \( p \) solves problem (19), (iii) agents have rational expectations; i.e.,
\( P = p \), (iv) the store is viable; i.e., \( \pi \geq 0 \), (v) the existing store prevents viable entry; i.e., there is no
\( \gamma' > \gamma \) yielding store profit \( \pi' \geq 0 \), (vi) the store attracts customers; i.e., \( \gamma \geq \epsilon \), (vii) the store maximizes
profit \( \pi \), given conditions (iv)-(vi), (viii) there is a determinant amount of money in circulation; i.e.,
\( 0 < M < 1 \), (ix) both the store and money mediate exchange; i.e., \( \gamma - \epsilon + \beta V_g = \beta V_M \).
Conditions (i)-(vii) are conditions on agent and store behavior. Condition (viii) indicates we are looking for an equilibrium where money circulates, and condition (ix) indicates we are looking for an equilibrium where money does not dominate the store as a medium of exchange, nor vice versa.

Lemma 1 (Excess Demand Function for Money): Let $\beta V_M - [\gamma - \varepsilon + \beta V_G]$ denote the excess demand for money. If $0 < M < 1$, and $\varepsilon \leq \gamma < 1$, then

$$
(24) \quad \beta V_M - [\gamma - \varepsilon + \beta V_G] = 0 \iff P - x = \frac{\gamma - \varepsilon}{\beta \theta x [1 - M \Gamma (1 - \gamma)]}
$$

Proof: See Appendix

Lemma 2 (Equilibrium Quantity of Money): In any equilibrium where $P \neq x$ and $\gamma \neq 1$,

$$
(25) \quad M = \frac{[1 - \gamma] \beta \theta [P - x] - [\gamma - \varepsilon]}{[1 - \gamma] \beta \theta [P - x] - [\gamma - \varepsilon]} = 1 - \frac{\gamma - \varepsilon}{[1 - \gamma] \beta \theta [P - x]}
$$

Proof: From Lemma 1, the equilibrium condition $\gamma + \beta V_G = \beta V_M$ implies $P - x = [\gamma - \varepsilon]/[\beta \theta x [1 - M \Gamma (1 - \gamma)]]$. Solving for $M$ then yields condition (25).

Lemma 3 (Equilibrium Trading Rate Restriction): In an equilibrium where $0 < M < 1$, the trading rate $\gamma$ must satisfy

$$
(26) \quad \varepsilon < \gamma < \frac{\beta \theta x [P - x] + \varepsilon}{1 + \beta \theta x [P - x]}
$$

Proof: Using the solution (25) to replace the quantity of money in the equilibrium condition $0 < M < 1$, solving for $\gamma$ directly yields condition (26).

Using Lemma 1, we can obtain a restriction on the willingness of agents to accept money that must hold if a monetary equilibrium is to exist.

Lemma 4 (Restriction on willingness to accept money): In an equilibrium where $0 < M < 1$, the probability of accepting money must satisfy

$$
(27) \quad P > x + \frac{\gamma - \varepsilon}{[1 - \gamma] \beta \theta x}.
$$
Proof: If $P \leq x + [\gamma - \varepsilon] [1 - \gamma] \beta \theta x$, then condition (25) indicates $M \leq 0$, which violates the money circulation condition $0 < M < 1$.

Lemma 4 indicates agents must have enough confidence in money in order for it to circulate in equilibrium. When this confidence exists, and money circulates, condition (25) allows us to understand how the store’s choice for $\gamma$ influences the quantity of money in circulation. As the store’s markup increases so that the trading rate $\gamma$ decreases toward the transactions cost $\varepsilon$, store trade becomes less attractive and money becomes the dominant medium of exchange; i.e., $M \to 1$. Alternatively, as the markup decreases and the trading $\gamma$ increases and approaches $[\beta \theta x [P - x] + \varepsilon] / [1 + \beta \theta x [P - x]]$, store trade becomes more attractive and money is driven out of the economy; i.e., $M \to 0$. The condition $[\beta \theta x [P - x] + \varepsilon] / [1 + \beta \theta x [P - x]]$ is increasing in $P$ and $\beta$. Thus, when there is more confidence in money, or when agents discount the future less, a higher trading rate $\gamma$ is necessary to drive money out of the economy.

Remember that, in the Kiyotaki and Wright Fiat Money Economy, $P > x$ had to hold for money to circulate. Condition (27) indicates, in our Money-Store Economy, agents must generally have even more confidence in money for it to circulate. This is because monetary trade does not just compete with the barter opportunity, but it must also compete with the store. The store is no competition when its markup is so high that $\gamma = \varepsilon$. However, as the store’s trading rate increases above this threshold level, agents must have increasing confidence in money for it to circulate while competing with the store.

Theorem 2 (Non-Existence of a Monetary Equilibrium for Fixed Cost Case): When store operation cost is the fixed cost $c$ and expectations are rational (i.e. $P = p$), no equilibrium exists with $\varepsilon \leq \gamma < 1$ for the Monetary Economy.

Proof: See Appendix

What we learn from Theorem 2 is, if a store of the fixed cost type can exist, it will drive money out of the economy. Increasing the trading rate reduces the store’s markup, which makes the store more attractive. This increases the number of traders through the store and reduces the number of money
holders. Profits increase because the fixed cost is spread over more traders, and the store becomes increasingly efficient. Eventually, the attractiveness of the store drives money out of the economy, so a monetary equilibrium cannot exist.

**Theorem 3 (Existence of a Monetary Equilibrium for Variable Cost Case):** When the variable store cost \( \hat{c} \) satisfies

\[
\frac{1-\varepsilon}{1+\beta\theta\varepsilon[1-x]} < \hat{c} < 1-\varepsilon,
\]

then a set of equilibria exists for the Monetary Economy with a store trading rate

\[
\gamma = 1 - \hat{c}.
\]

The set of equilibria contains an infinite number of elements, parameterized by the equilibrium expectation \( P^* \), where and \( P^* \) satisfies

\[
\overline{P} \equiv x + \frac{1-\varepsilon - \hat{c}}{\beta\theta\varepsilon} < P^* \leq 1,
\]

and the equilibrium quantity of money \( M^* \) satisfies

\[
M^* = 1 - \frac{1-\varepsilon - \hat{c}}{\beta\theta\varepsilon[P^*-x]}.
\]

**Proof:** See Appendix

From conditions (29) and (31), we see that, as the per unit store operation cost \( \hat{c} \) increases to \( 1-\varepsilon \), the equilibrium trading rate \( \gamma \) decreases to \( \varepsilon \) and economy’s money supply \( M \) increases to 1. That is, money becomes a more dominant medium of exchange as the cost of operating the store increases. As the cost \( \hat{c} \) decreases to \( [1-\varepsilon]/[1+\beta\theta\varepsilon[1-x]] \), the expectation \( P^* \) must increase to 1 for a monetary equilibrium to exist, trading rate \( \gamma \) increases to \( [\beta\theta\varepsilon[1-x] + \varepsilon]/[1+\beta\theta\varepsilon[1-x]] \) and the money in circulation \( M \) decreases to zero.

The threshold level for money confidence is \( \overline{P} = x + [(1-\varepsilon - \hat{c})]/(\hat{c} \beta\theta\varepsilon) \) in condition (30), so there are multiple monetary equilibria associated with expectations in the range \( P \in (\overline{P},1] \). As the store
operation cost $\hat{c}$ increases, the expectation $P^*$ can be lower with money still circulating, and the low
money confidence bound $\overline{P} = x$ is reached as $\hat{c}$ reaches $1 - \varepsilon$. As $x$ increases, agents must have
more confidence in money for it to exist, and there is some value for $x$ large enough that no store can
exist and support the circulation of money, because barter becomes so effective. Condition (30)
indicates that if people discount the future enough ($\beta$ small enough), if the store operations cost is low
enough ($\hat{c}$ small enough), if a match in the social interaction is too unlikely ($\theta$ small enough), or a single
coincidence of wants too unlikely ($x$ small enough), then a monetary equilibrium cannot be support even
if agents are entirely confident that money will be accepted (i.e., $P = 1$).

**Theorem 4 (Stability of Monetary Equilibrium---Variable Cost Case):** Let $M^*$ denote the equilibrium
quantity of money, where $\beta V_M = \gamma - \varepsilon + \beta V_G$. For all $M \in \left[0, M^*\right)$, $\beta V_M > \gamma - \varepsilon + \beta V_G$, and for all
$M \in \left(M^*, 1\right]$, $\beta V_M < \gamma - \varepsilon + \beta V_G$.

**Proof:** See Appendix

Theorem 4 is intuitive. When there is no money in circulation, or very little, accepting money
offers the largest advantage it can offer, for the probability of being matched with a good holder who will
accept the money in exchange is as high as it can be. If holding money is better than holding good in this
situation, then good holders will exchange good for money at any opportunity, and the quantity of money
will increase. As the quantity of $M$ increases, there are fewer good holders for a money holder to be
matched with, so the expected value associated with holding money decreases. Assumption (28) ensures
there can be so much money that its expected value is less than that associated with holding good. In this
high money quantity situation, money holders will exchange good for money at any opportunity, and the
quantity of money decreases toward the equilibrium quantity.

6. Welfare
To examine welfare, we use the criterion \( W = MV_M + (1-M)V_G. \) In equilibrium, when
\[
\beta V_M = \gamma - \epsilon + \beta V_G, \text{ the value functions (18) and (20) reduce to } V_M = \big[1-M\big]\partial x P[1-\gamma] / [1-\beta] \text{ and }
\]
\[
V_G = \big[1-M\big]\partial x^2 [1-\gamma] + \big[\gamma - \epsilon\big] / [1-\beta], \text{ so we can write the welfare function as }
\]
\[
(32) \quad W = \frac{M[1-M] \partial x P[1-\gamma]}{1-\beta} + \frac{[1-M]\partial x^2 [1-\gamma] + \gamma - \epsilon}{1-\beta}.
\]

Note that, if a monetary economy cannot form so \( M = 0, \) the welfare function reduces to \( W = V_G, \) where the welfare level \( V_G \) in condition (32) is the same as that for the Store Economy in condition (11). Because money holding precludes production and reduces the likelihood of barter, we see in condition (32) that the portion of welfare associated with good holding is monotonically decreasing in \( M. \) The portion of condition (32) associated with money holding indicates that more money contributes to welfare when the quantity of money is low, but detracts from welfare when the quantity of money is high. An increase in the money level increases the fraction of agents who reap the reward of trading money for good, but more money reduces production and reduces the probability of a money match. The former effect dominates when the money level is low, but the latter effect dominates when the money level is high.

The effect of the money level on welfare is given by the derivative
\[
(33) \quad \frac{dW}{dM} = \frac{\partial x[1-\gamma]}{1-\beta} \big[P - 2x - 2[P - x]M\big] - \frac{\gamma - \epsilon}{1-\beta}.
\]

Evaluating at \( M = 0, \) with \( \gamma = 1 - \hat{c}, \) we find that the introduction of money to the Store Economy can increase welfare only if
\[
(34) \quad P > \tilde{P} = 2x + \frac{1 - \hat{c} - \epsilon}{\partial x \hat{c}}.
\]

When \( P = 1, \) as in the monetary equilibrium for the Kiyotaki and Wright Fiat Money Economy, and \( \hat{c} = 1 - \epsilon \) is imposed to rule out the existence of the store, then condition (34) reduces to
\( x < 1/2 \), which is the condition Ljungqvist and Sargent (2000) show must hold in order for fiat money to be welfare improving in the fiat money economy.

For the Store-Money Economy, when money endogenously arises and finds its equilibrium value, the equilibrium quantity of money will be welfare maximizing only by chance. The optimal quantity of money, found by setting the derivative (33) equal to zero, can be written as

\[
\hat{M} = 1 - \frac{\beta [1 - \hat{c} - \beta]}{2\theta \hat{x}\hat{c}^2 [P - x]} - \frac{P}{2[P - x]}
\]

Comparing this optimal quantity of money to the equilibrium quantity of money \( M^* \) given by (31), we find

\[
M^* \begin{cases} > \hat{M} \iff P^* = 1 - \hat{c} - \epsilon \frac{2 - \beta}{\beta \hat{x}} \end{cases}
\]

That is, we find that, when an interior optimum exists for the quantity of money, there exists a unique confidence level \( \hat{P} \) that will bring forth the optimal quantity. Full confidence in fiat money (i.e., \( P = 1 \)), or any confidence level \( P > \hat{P} \), brings forth more money than is optimal, while \( P < \hat{P} \) brings forth too little. The following theorem delineates how the introduction of money affects welfare, in general.

*Theorem 5 (Welfare)*: For any monetary equilibrium, when \( x < [1 - \beta] \frac{1 - \epsilon}{1 + \beta \hat{x}[1 - x]} \), the effect of an increase in the quantity of money affects welfare depends upon the store’s operation cost:

**Case 1:**

\[
\frac{1 - \epsilon}{1 + \beta \hat{x}[1 - x]} < \hat{c} < \frac{2 - \beta \frac{1 - \epsilon}{1 - \beta}}{2 - \beta \frac{1 - \epsilon}{1 - \beta}} \iff \frac{\partial W}{\partial M^*} > 0 \text{ for all } P^* \in [\hat{P}, 1]
\]

**Case 2:**

\[
\frac{2 - \beta \frac{1 - \epsilon}{1 - \beta}}{2 - \beta} + \beta \hat{x} \times \hat{c} < \frac{1 - \beta \frac{1 - \epsilon}{1 - \beta}}{1 - \beta} + \beta \hat{x} \times \hat{c}^2 \iff \frac{\partial W}{\partial M^*} > 0 \text{ for all } P^* \in [\hat{P}, \hat{P}]
\]

\[
\frac{\partial W}{\partial M^*} < 0 \text{ for all } P^* \in [\hat{P}, \hat{P}]
\]
Case 3: \[\frac{[1 - \beta][1 - \varepsilon]}{[1 - \beta] + \beta \theta x^2} < \hat{c} < 1 - \varepsilon \iff \frac{\partial W}{\partial M^*} < 0 \text{ for all } P^* \in [\bar{P}, 1].\]

However, when \[x \geq \frac{[1 - \beta]/[1 + [1 - \beta]]}{1 + \beta \theta x[1 - x]} \] an increase in the quantity of money only reduces welfare in that \[\frac{[1 - \varepsilon]}{1 + \beta \theta x[1 - x]} < \hat{c} < 1 - \varepsilon \iff \frac{\partial W}{\partial M^*} < 0 \text{ for all } P^* \in [\bar{P}, 1].\]

Proof: See Appendix

Theorem 5 indicates that, in order for the introduction of money to be welfare enhancing, agents must discount the future enough and barter cannot be too easy. When barter is difficult enough or when agents discount the future enough, the level of the store’s operation cost determines whether or not more money enhances welfare. When the store has a low operation cost (i.e., Case 1), more money only increases welfare, and more money is present in the economy as the confidence agents have in money increases from \(\hat{P}\) to 1. When the store operation cost is high (i.e., Case 3), more confidence in money only decreases welfare. The intermediate case (Case 2), where there is an interior optimum for the quantity of money, occurs when the store cost is in an intermediate range. In this case, regarding the impact on welfare, agents can have either too much confidence in money (i.e., \(P^* > \hat{P}\)) or too little (i.e., \(P^* < \hat{P}\)).

7. Discussion

We have presented a model in which fiat money naturally arises from the competition for a contestable mediation market. The equilibrium quantity of fiat money is determined by the microfoundation of the model, not by government. As in the traditional search theoretic models of fiat money, the confidence agents have in money is important for determining whether the circulation of fiat money can be supported. However, in our model, for a given confidence level \(P\), the equilibrium quantity of money \(M^*\) is stable, meaning there are market forces that will
drive the quantity of money back to $M^*$ if the quantity is not at $M^*$. This has implications for government monetary policy.

For example, suppose government introduces a form of fiat money before our store recognizes it can enhance its hold on the mediation market by introducing money. Assume agents view government money as equivalent to that introduced by the store in our model. Our model indicates government could introduce a quantity of money up to the equilibrium amount of money $M^*$ for our model, and in doing so would crowd out the ability of the store to introduce money. However, government could not get agents to voluntarily accept an amount of money above $M^*$ because agents would prefer to trade their good to the store in exchange for good. In general, our model suggests that recognizing competing mediums of exchange is important for evaluating the scope of money’s impact.

In addition, our model indicates taxes on trade can impact the quantity of money in circulation and, consequently, the level of welfare. The equilibrium quantity of money $M^*$ in our model is almost surely not optimal. Condition (31) indicates that the implementation of a transaction tax that effectively increases the transactions cost $\varepsilon$ would increase the equilibrium quantity of money, providing an increase in welfare if the original equilibrium quantity was too low. From the discussion in the previous paragraph, we know a government attempt to increase the money supply would not accomplish this same improvement in welfare. This suggests further exploration of models with competing mediums of exchange may enhance the understanding of how welfare may be affected by the interaction of monetary and fiscal policies.

Finally, the simplicity of the model we have presented appears to make it useful for exploring a variety of institutional and policy issues. For example, it would seem that banking could readily be introduced in that our model has characteristics similar to that of He, Huang,
and Wright (2005). In fact, we may reinterpret our model so that it has a banking sector. The store’s issuance of IOUs can be interpreted as the agent making a deposit in a bank after selling good to the store. Money is created when the bank credits the account of the agent for this deposit. As our model is formulated here, the bank never makes loans, but holds 100 percent of deposits as reserves and only facilitates transactions by delivering the purchasing power when our money holder makes a trade for good. Because many agents holding money will not experience the single coincidence of wants necessary to make a trade, some bank reserves will go unused. By modeling bank loans made from these excess reserves, it appears that the impact of fractional reserve banking could be examined. Following He, Huang, and Wright (2005) and introducing the assumption that money is subject to theft, but not bank deposits, it may also be possible to distinguish coin and currency circulation from demand deposits.

Appendix: Proofs

**Proof of Theorem 1:** Because profits are continuously decreasing in \( \gamma \), any level for \( \gamma \) that generates a positive profit level cannot be sustained, for a viable competing store can be created with \( \gamma' > \gamma \), so all agents will prefer the competing store. Thus, to prevent the successful entry of a competing store, the existing store must adjust \( \gamma \) to eliminate profit. In the fixed cost case, the store earns zero profit when 
\[
[1 - \gamma] \left[ 1 - \theta \bar{c}^2 \right] - \bar{c} = 0, \quad \text{or when} \quad \gamma = \left[ \frac{1 - \theta \bar{c}^2}{1 - \theta \bar{c}^2} \right] = 1 - \bar{c} / \left[ 1 - \theta \bar{c}^2 \right].
\]
Given this value for \( \gamma \), the store attracts customers if and only if \( \varepsilon \gamma \geq 0 \), which implies \( \bar{c} \leq \left[ 1 - \theta \bar{c}^2 \right] / \left[ 1 - \varepsilon \right] \). In the variable cost case, the store earns zero profit when 
\[
[1 - \gamma] \left[ 1 - \theta \hat{c}^2 \right] = 0, \quad \text{or when} \quad \gamma = 1 - \hat{c}.
\]
Given this value for \( \gamma \), the store attracts customers if and only if \( \varepsilon \gamma \geq 0 \), which implies \( \hat{c} \leq 1 - \varepsilon \).

**Proof of Lemma 1:** Let \( p^* \) denote the optimal value for \( p \) in problem (19). If \( \gamma \geq \varepsilon \), then the condition
\[
\max \left\{ \max \{ p \beta V_M + [1 - p] (\gamma - \varepsilon + \beta V_G) \}, \beta V_G \right\}
\]
in value function (20) becomes
\[
p^* V_M + [1 - p^*] (\gamma - \varepsilon + \beta V_G),
\]
for no agent will choose to hold good from one period to the next. Using the value functions (18) and (20) to construct the difference 
\[
D = \beta V_M - (\gamma - \varepsilon + \beta V_G),
\]
we can write
\[
D = \beta \theta x \left[ 1 - M \right] (P - x) \left[ (1 - \varepsilon + \beta V_G) - [p^* \beta V_M + (1 - p^*) (\gamma - \varepsilon + \beta V_G)] \right] - (\gamma - \varepsilon).
\]
Letting \( A \) denote the quantity \( \beta \theta x \left[ 1 - M \right] (P - x) \), the last condition can be rewritten as
\[
D = A \left[ 1 - \gamma - p^* D \right] - (\gamma - \varepsilon),
\]
which implies 
\[
[1 + p^* A] D = A \left[ 1 - \gamma \right] - [\gamma - \varepsilon].
\]
Because
\[
0 < \beta \theta x \left[ 1 - M \right] < 1 \quad \text{and} \quad -1 < P - x < 1,
\]
we know 
\[
-1 < A < 1.
\]
Knowing \( A > -1 \) and \( 0 \leq p^* \leq 1 \),

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we know \[ 1 + p^*A > 0 \]. Thus, \[ 1 + p^*A D = A[1 - \gamma] - [\gamma - \varepsilon] \] implies the sign of \( D \) depends upon the sign of \( A[1 - \gamma] - [\gamma - \varepsilon] \). Condition (24) follows directly.

**Proof of Theorem 2:** By Lemma 2, the quantity of money in equilibrium must satisfy condition (25).
Using this quantity of money, the profit function for the store in the fixed cost case becomes

\[
\pi = \frac{\gamma - \varepsilon}{\beta \theta [1 - \gamma]} \left[ 1 - p \theta \right] + \left[ \frac{\gamma - \varepsilon}{\beta [1 - \gamma]} \right][p - x] - \bar{c}
\]

From Lemma 2, we know \( P - x > 0 \) must hold for an equilibrium to exist with \( \varepsilon \leq \gamma < 1 \). Under rational expectations \( p - x > 0 \) must also hold. It then follows that the store’s profit is strictly increasing in the trading rate \( \gamma \). Any store would therefore be motivated to increase \( \gamma \) to a level such that \( \gamma > \left[ \beta \theta [1 - \gamma] + \varepsilon \right][1 + \beta \theta [P - x]] \), violating condition (26) of Lemma 3. The contestable market will not prevent this increase in the trading rate, but rather encourage it. This is sufficient to complete the proof.

**Proof of Theorem 3:** Together, the dynamic program conditions (18) and (20) and the equilibrium condition \( \gamma - \varepsilon + \beta V_G = \beta V_M \) imply that the quantity of money in equilibrium must satisfy condition (25). Using this quantity of money, the profit function for the store in the variable cost case becomes

\[
\pi = [1 - \gamma - \hat{c}] \left[ \frac{\gamma - \varepsilon}{\beta \theta [1 - \gamma]} \right][1 - p \theta \left[1 - \gamma] + \left[ \frac{\gamma - \varepsilon}{\beta [1 - \gamma]} \right][p - x] - \bar{c}
\]

Profit per store customer, \( 1 - \gamma - \hat{c} \), is decreasing in \( \gamma \). Because \( P > x \) and \( P = p \) by the rational expectations assumption, the number of store customers,

\[
\left[ \frac{\gamma - \varepsilon}{\beta \theta [1 - \gamma]} \right][1 - p \theta \left[1 - \gamma] + \left[ \frac{\gamma - \varepsilon}{\beta [1 - \gamma]} \right][p - x] - \bar{c}
\]

is increasing in trading rate \( \gamma \). By setting \( \gamma = \varepsilon \), the store obtains a high profit per unit, but has no customers. As \( \gamma \) increases from \( \varepsilon \) to \( 1 - \hat{c} \), the number of customers continuously increases, while the profit per unit decreases continuously to zero. Therefore, a profit maximum exists in this domain for \( \gamma \). Contestability implies the trading rate must be set at \( \gamma = 1 - \hat{c} \) to eliminate the profit that would entice competing stores to enter. With \( \gamma = 1 - \hat{c} \), condition (25) implies the money in circulation is the amount presented as condition (31). Because \( \gamma - \varepsilon + \beta V_G = \beta V_M \), agents are indifferent between holding money and trading through the store, so agents are indifferent about the value of \( p \). Thus, with \( \gamma = 1 - \hat{c} \), any value for the expectation \( P \) that supports the circulation of a positive quantity of money can qualify as an equilibrium expectation. The restriction (28) ensures that there exists at least one such value for \( P \). Condition (30) gives the range of values for \( P \) that support a money supply \( M \) such that \( 0 < M < 1 \), and condition (31) gives the equilibrium money supply. Because all equilibrium conditions are satisfied for values of \( P \) satisfying condition (30), the proof is complete.

**Proof of Theorem 4:** Let \( Z = [1 - \hat{c} - \varepsilon] / [\beta \theta [1 - M] \hat{c}] \). Because \( \hat{c} < 1 - \varepsilon \) by assumption (28), \( Z \) is monotonically increasing in \( M \) for all \( M \in [0,1] \). Because \( \gamma = 1 - \hat{c} \) and \( \beta V_M = \gamma - \varepsilon + \beta V_G \) in
equilibrium, condition (24) implies \( P - x = Z^* \), where \( Z^* = [1 - \hat{c} - \varepsilon] / [\beta \theta x [1 - M^*]^2] \). Because \( Z \) is increasing in \( M \), it follows that \( P < x + Z \), for all \( M \in (M^*, 1) \), and condition (24) implies \( \beta V_M < \gamma - \varepsilon + \beta V_G \). Because \( Z \) is increasing in \( M \), it also follows that \( P > x + Z \), for all \( M \in [0, M^*) \), and condition (24) implies \( \beta V_M > \gamma - \varepsilon + \beta V_G \).

Proof of Theorem 5: From (31), we know the equilibrium quantity of money \( M^* \) is strictly increases from zero to a quantity we will call \( M^*_{\text{max}} \) as \( P \) increases over the interval \( P \in [\bar{P}, 1] \). The second derivative of the welfare function (32) is \( \partial^2 W / \partial M^2 = -[2 \theta \hat{c} [P - x] / [1 - \beta] \], which is strictly negative for all \( M \in [0, 1) \) when \( P > x \). Since \( P > x \) in any monetary equilibrium, this negative second derivative implies we have three possible cases. As \( P \) increases over the interval \( P \in [\bar{P}, 1] \), so that \( M^* \) increases over the interval \( M^* \in [0, M^*_{\text{max}}] \), welfare either strictly increases (Case 1), strictly increases to a peak and then strictly decreases (Case 2), or strictly decreases (Case 3).

From (36), we know Case 1 occurs when \( \hat{P} > 1 \) because \( P < \hat{P} \) must hold for all \( P \in [\bar{P}, 1] \), which implies \( M^* < \hat{M} \) for all \( M^* \in [0, M^*_{\text{max}}] \). Using the definition of \( \hat{P} \) in (36), \( \hat{P} > 1 \) implies \( \hat{c} < \left[ 2 - \beta [1 - \varepsilon] / [2 - \beta] + \beta \theta x \right] \). In order for \( M^* \) to be an equilibrium value, (28) implies \( \hat{c} > [1 - \varepsilon] / [1 + \beta \theta x [1 - x]] \). These last two inequalities hold simultaneously if and only if \( x < [1 - \beta] / [1 + [1 - \beta]] \). Thus, when \( x < [1 - \beta] / [1 + [1 - \beta]] \), Case 1 holds if and only if \( [1 - \varepsilon] / [1 + \beta \theta x [1 - x]] < \hat{c} < \left[ 2 - \beta [1 - \varepsilon] / [2 - \beta] + \beta \theta x \right] \).

From (36), we know Case 2 occurs when \( \bar{P} < \hat{P} < 1 \). This is because \( P < \hat{P} \) must hold for all \( P \in [\bar{P}, \hat{P}] \), which implies \( M^* < \hat{M} \) for all \( M^* \in [0, \hat{M}] \), but then \( P > \hat{P} \) must hold for all \( P \in (\hat{P}, 1] \), which implies \( M^* > \hat{M} \) for all \( M^* \in (\hat{M}, M^*_{\text{max}}) \). Using the definitions of \( \bar{P} \) and \( \hat{P} \) in (30) and (36), \( \hat{P} > \bar{P} \) implies \( \hat{c} < \left[ 1 - \beta [1 - \varepsilon] / [2 - \beta] + \beta \theta x^2 \right] \). Using the definition of \( \hat{P} \) in (36), \( \hat{P} < 1 \) implies \( \hat{c} > \left[ 2 - \beta [1 - \varepsilon] / [2 - \beta] + \beta \theta x \right] \). These last two inequalities hold simultaneously if and only if \( x < [1 - \beta] / [1 + [1 - \beta]] \). Thus, when \( x < [1 - \beta] / [1 + [1 - \beta]] \), Case 2 holds if and only if \( \left[ 2 - \beta [1 - \varepsilon] / [2 - \beta] + \beta \theta x \right] < \hat{c} < \left[ 1 - \beta [1 - \varepsilon] / [1 - \beta] + \beta \theta x^2 \right] \).

From (36), we know Case 3 occurs when \( \hat{P} < \bar{P} \) because \( P > \hat{P} \) must hold for all \( P \in [\bar{P}, 1] \), which implies \( M^* > \hat{M} \) for all \( M^* \in [0, M^*_{\text{max}}] \). Using the definitions of \( \bar{P} \) and \( \hat{P} \) in (30) and (36), \( \hat{P} < \bar{P} \) implies \( \hat{c} > \left[ [1 - \beta [1 - \varepsilon] / [1 - \beta] + \beta \theta x^2 \right] \). The quantity \( \left[ 1 - \beta [1 - \varepsilon] / [1 - \beta] + \beta \theta x^2 \right] \) is strictly less than \( 1 - \varepsilon \) for all the admissible parameter values \( 0 < \beta < 1, 0 < x < 1, 0 < \varepsilon < 1 \), and \( 0 < \theta < 1 \). Thus, when \( x < [1 - \beta] / [1 + [1 - \beta]] \), Case 3 holds if and only if \( \left[ 1 - \beta [1 - \varepsilon] / [1 - \beta] + \beta \theta x^2 \right] < \hat{c} < 1 - \varepsilon \).
When $x = [1 - \beta]/[1 + [1 - \beta]]$, 
$[[1 - \varepsilon]]/[1 + \beta\alpha[1 - x]] = [[1 - \beta][1 - \varepsilon]]/[1 - \beta] + \beta\alpha x^2 = [[2 - \beta][1 - \varepsilon]]/[2 - \beta] + \beta\alpha x$, so it is not possible to find a value for $\hat{c}$ such that satisfies either Case 1 or Case 2. However, we may still find a value for $\hat{c}$ such that $[[1 - \beta][1 - \varepsilon]]/[1 - \beta] + \beta\alpha x^2 < \hat{c} < 1 - \varepsilon$, so Case 3 may still hold when $x = [1 - \beta]/[1 + [1 - \beta]]$. When $x > [1 - \beta]/[1 + [1 - \beta]]$, 
$[[2 - \beta][1 - \varepsilon]]/[2 - \beta] + \beta\alpha x < [1 - \varepsilon]/[1 + \beta\alpha[1 - x]]$ and 
$[[1 - \beta][1 - \varepsilon]]/[1 - \beta] + \beta\alpha x^2 < [[2 - \beta][1 - \varepsilon]]/[2 - \beta] + \beta\alpha x$. Thus, it is not possible to find a value for $\hat{c}$ such that satisfies either Case 1 or Case 2. However, we may still find a value for $\hat{c}$ such that $[[1 - \beta][1 - \varepsilon]]/[1 - \beta] + \beta\alpha x^2 < \hat{c} < 1 - \varepsilon$, so Case 3 may still hold when $x > [1 - \beta]/[1 + [1 - \beta]]$. Thus, we can conclude that when $x \geq [1 - \beta]/[1 + [1 - \beta]]$, only Case 3 may hold, which is sufficient to complete the proof.
References


