A Semi-Empirical Model for Porous Media Heat Exchanger Design

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Abstract

A semi-empirical model for conduction/convection in a thin “fin-like” porous wall of arbitrary composition and thickness is developed. The model assumes one-dimensional conduction in the porous matrix and one-dimensional flow of the coolant through the wall.

Closed form solutions are obtained for a uniform composition and thickness porous wall having an insulated tip. Analysis shows that the equivalent unit surface conductance and the effectiveness of the porous wall are always maximum when it is operated with the number of transfer units of the porous matrix greater than two (ntu ≥ 2). Furthermore, if the porous matrix is composed of a packed bed of spherical particles, it is found that, for a given coolant flow rate, the pressure drop across the porous matrix is minimum when ntu ≤ 2. This suggest that for many design requirements, the porous media exchanger should be operated at ntu = 2.

Nomenclature

A porous wall cross section area, tW
As heat transfer surface area
cp coolant specific heat
d particle diameter
F see Eq. (12)
G coolant mass velocity
h unit surface conductance
H porous wall height
ke porous media effective conductivity
k_c coolant conductivity
mH see Eq. (13)

ntu porous media number of transfer units
p pressure
Pr Prandtl number
q conduction heat transfer rate
Re particle Reynolds number
ReH coolant Reynolds number
St particle Stanton number
t porous wall thickness
T temperature
U equivalent unit surface conductance
Vol volume
W porous wall width
y coordinate
β heat transfer surface area-to-volume ratio
e porosity
ρ coolant density
μ coolant viscosity
Θ non-dimensional temperature

Introduction

Compact heat exchangers invariably incorporate heat transfer augmentation technology at the fluid-solid interface. The more popular and well documented passive techniques summarized by Webb [1994] include: roughened walls, extended and modified surfaces, and displaced inserts. Roughness modifies the viscous sub-layer, and as a consequence affects the unit surface conductance, h. It is most effective at high (turbulent flow) Reynolds numbers. Extended surfaces (fins) primarily increase the heat transfer surface area, although some
fin configurations also increase h. For example, strip-fins interrupt thermal boundary layer growth, leading to higher values of h. Inserts (for example, twisted tapes) modify the core flow in such a way as to increase transport at the wall. Geometric modification of the heat transfer passage walls may also accomplish this. Wavy-wall channels promote swirl flow, leading to augmentation [Snyder et al., 1993]. Transverse grooves destabilize Tollmien-Schlichting waves, leading to early transition and increased mixing [Greiner et al., 1990].

The use of a conductive porous media as the exchange matrix at the fluid-solid interface of a compact heat exchanger will also enhance performance. The technique offers several advantages. These include: a very high heat transfer surface area to volume ratio; easily fabricated and inexpensive, yet potentially complex, exchange matrix shapes; and the opportunity to tailor the h-Δp characteristics of the surface to a specific requirement.

One measure of a high-performance heat exchange matrix is its heat transfer surface area-to-volume ratio, \( \beta = A_s/\text{Vol} \). For example, a parallel plate heat exchanger has \( \beta_{\text{pl}} = 1/H \), where 2H is the plate spacing. When \( H = 1 \text{ mm} \), \( \beta_{\text{pl}} = 1000 \text{ m}^{-1} \). Off-set strip-fin surface #18.986 has \( \beta_{\text{off-set}} = 2254 \text{ m}^{-1} \) [Kays and Crawford, 1993]. By comparison, a porous matrix consisting of unconsolidated spherical particles of diameter, d, and porosity, \( \epsilon \), has [Wakao and Kaguei, 1982]

\[
\beta_{\text{porous}} = 6(1-\epsilon)/d \tag{1}
\]

A typical value for the present application, with \( d = 1 \text{ mm} \) and \( \epsilon = 0.4 \), is \( \beta_{\text{porous}} = 3600 \text{m}^{-1} \). For the same heat exchanger volume, the porous matrix provides approximately 1.5 times more heat transfer surface than the off-set strip-fin array, and 3.6 times more than the un-enhanced plate surface. The use of smaller particles or a material having a lower porosity will result in even more favorable ratios.

The thermal conductance of a heat exchange surface is proportional to the product of the unit surface conductance of the fluid-solid interface and the heat transfer surface area, \( hA_s \). At fixed coolant flow rate and heat exchanger frontal area, h will be approximately the same for different surface configurations. Therefore, significant increases in \( A_s \) translate directly to either an increase in capacity or a reduction in exchange size and weight.

A porous exchange matrix can be formed by gravity sintering metallic particles. Gravity sintering is essentially a casting process. Metallic particles are shaped (usually via tumbling), pre-tinned with a eutectic and flux agent, poured in a mold and heated to a temperature above the eutectic. The bonded piece is then removed from the mold. Innovative mold design will allow for highly complex yet inexpensive exchanger shapes. The process has been used for decades in the production of metallic filters and flame holders. More recently it has been used to form the wick for a flexible heat pipe used to cool the microprocessor of a laptop computer [Rosenfeld et al., 1996] and some specialized heat exchanger cores. Liquid-coolers, having capacities exceeding 5000 watt/cm², have been developed to cool the Gyrotrons in a fusion reactor [Zapevolov et al., 1991], and Lindemuth et al. [1994] report developing a laser cavity cooler having a capacity in excess of 6000 watt/cm².

This paper develops a semi-empirical model for heat transfer in a thin, porous, fin-like wall of arbitrary composition and thickness. Part of the present model (the thermodynamic-limit solution described below) is similar to a heat transfer model previously developed by Rosenfeld [1992]. Solutions for walls having uniform composition and thickness are described. Criteria for optimizing the exchange matrix performance characteristics are identified, and a design procedure for incorporating a porous exchange matrix in overall system design is outlined.

**Heat Transfer Model**

Consider a thin, porous, fin-like wall attached to a plane surface as shown in Fig. 1. The wall has height, \( H \), depth, \( W \), and variable thickness, \( t(y) \), where \( y \) is the vertical coordinate. The physical properties of the porous wall (\( \epsilon, \beta, \) and composition) are assumed to vary only with coordinate \( y \). The temperature of the wall is \( T(y) \), and it is cooled by a coolant that flows through it. The coolant, at uniform temperature \( T_i \) and pressure, \( p_i \), approaches the wall with mass velocity, \( G_i(y) \) [kg/s per m²]. It exits the wall at uniform pressure, \( p_o \), temperature, \( T_o(y) \) and mass velocity, \( G_o(y) \).

**Coolant flow distribution.** Assume the coolant flow within the porous matrix is approximately one-dimensional. Then, \( G_o(y) = G(y) \) and the volume average mass and momentum equations reduce to an Ergun-type expression relating pressure drop to mass velocity [Vafai and Sozen, 1990]. This relation can be used to establish the function \( G(y) \) for given pressure drop, physical properties of the wall, and coolant density and viscosity. For example, if the wall consists of a matrix of spherical particles, then the appropriate correlation, expressed in terms of a loss coefficient, \( C_l \), is [Bird et al., 1960].
\[
C_f \equiv \frac{p_t - p_o}{\rho \left( \frac{G_o}{\rho} \right)^2} = \frac{(1 - \varepsilon) t}{\varepsilon^3 \rho} \left[ \frac{150(1 - \varepsilon)}{\text{Re}} + 1.75 \right] \tag{2}
\]

where \( \text{Re} = G_o d / \mu \) is the particle Reynolds number and, \( \rho \) and \( \mu \) are the coolant density and viscosity, respectively.

Since \( t, d, \) and possibly \( \varepsilon \) can be made to vary with position, the coolant mass velocity distribution, and hence the heat transfer characteristics of the wall can be tailored to a specific application. Figure 2 shows examples of straight tapered porous walls symmetrically placed between parallel plates spaced \( 2H \) apart. Note, taper \( < 0 \) results in the hour-glass shape shown in the left part of the figure while taper \( > 0 \) gives the convex shape shown on the right.

Figure 3 plots the normalized air mass velocity functions corresponding to taper \( = -0.8 \) and \( +0.8 \) for a wall fabricated from 1 mm diameter spherical particles with \( t(0) = 5 \text{ mm}, \varepsilon = 0.4 \) and \( \Delta p = 1 \text{ in. H}_2\text{O} \). In the figure, \( G_o(y) \) is normalized by the average mass velocity,

\[
\overline{G_o} = \frac{1}{H} \int_0^H G_o(y)dy \tag{3}
\]

The figure shows that the coolant flow distribution, and hence the heat transfer characteristics of the wall, can be tailored by proper selection in the wall thickness profile. It is noted that variations in \( t(0) \), and the uniform values of \( d \) and \( \varepsilon \) have only a small impact on the shape of the \( G_o(y) \) functions shown in the figure. It is also interesting that a stratification in particle size, \( d(y) \), could be used to alter the coolant flow distribution through the wall in a way similar to that shown in the figure. Furthermore, the coolant flow distribution can be unintentionally modified when the coolant density and viscosity are strongly temperature dependent. Porous media heat exchanger design algorithms employing hydrocarbon-based liquids as coolants should incorporate provision for accounting for temperature dependent properties.

**Porous wall conduction equation.** The one-dimensional conduction heat transfer rate in the wall is given by

\[
q = -k_e(y) t(y) W \frac{dT}{dy} \tag{4}
\]

where \( k_e(y) \) is the effective thermal conductivity of the porous matrix [Duncan et al., 1989]. Application of Eq. (4) to an energy balance on a thin slice of wall gives

\[
\left[ k_e(t(T - T_i)) \right] + c_p G_o(T_o - T_i) = 0 \tag{5}
\]

where the prime represents differentiation with respect to \( y \). An additional equation, relating the coolant temperature, \( T_o \), to the porous matrix temperature, \( T \), is needed to close the system. The one-dimensional model presents two alternatives:

1. **Convection-limit solution.** Heat transfer between the coolant and porous media is governed and limited by Newton’s cooling law, and

2. **Thermodynamic-limit solution.** The coolant is heated to the porous media temperature before exiting the matrix (i.e. local thermal equilibrium is attained).

**Convection-limit solution.** Application of Newton’s
cooling law to an energy balance on a thin slice of the wall gives

$$\left(T_o - T_i\right) = \frac{h_p Bt}{c_p G_o} \left(T - T_m\right) \tag{6}$$

where $h_p$ is the particle heat transfer coefficient and $T_m(y)$ is the temperature of the coolant as it passes through the porous matrix. Assume that

$$T_m = \frac{T_o + T_i}{2} \tag{7}$$

Then, Eq. (6) may be written as

$$\left(T_o - T_i\right) = \frac{2ntu}{2 + ntu} \left(T - T_i\right)_{ntu \geq 2} \tag{8}$$

where $ntu(y)$ is the local number of transfer units of the porous matrix,

$$ntu = St \cdot Bt \tag{9}$$

where $St = h_p / c_p G_o$ is the particle Stanton number. See correlations for flow through porous matrices formed from spherical particles are reviewed by Wakao and Kaguei [1982].

Since $T_m(y) \leq T(y)$, equation (8) indicates that the convection-limited solution is confined to situations where $ntu \leq 2$. Since the particle Stanton number generally decreases with increasing Reynolds number, this condition corresponds to high coolant mass velocity or thin-wall situations.

**Thermodynamic-limit solution ($ntu > 2$).** If the mass velocity of the coolant is relatively low, or the porous wall is relatively thick, the coolant temperature will approach the porous matrix temperature before exiting the wall. Within the context of the one-dimensional model, this gives

$$\left(T_o - T_i\right) = \left(T - T_i\right)_{ntu \geq 2} \tag{10}$$

**Non-Dimensional Equations.** Equations (5), (8) and (10) can be put in the following compact form

$$\left(\hat{k} \cdot \hat{\Theta}\right)' + \left(mHF\right)^2 \Theta = 0 \tag{11}$$

where

$$F = \begin{cases} \frac{2ntu}{2 + ntu}, & ntu \leq 2 \\ 1, & ntu > 2 \end{cases} \tag{12}$$

and

$$mH = \sqrt{\frac{c_p G_o H^2}{k_e(0)t(0)}} \tag{13}$$

In Eq. (11) the prime represents differentiation with respect to the scaled vertical coordinate, $\hat{y} = y / H$, and

$$\Theta = (T - T_i) / (T(0) - T_i)$$

is the non-dimensional porous wall temperature. The quantity $(mH)^2$ can be viewed as a non-dimensional mass velocity; and, $\hat{k}_e = k_e(y) / k_e(0)$ and $\hat{t} = t(y) / t(0)$ are the non-dimensional effective thermal conductivity and wall thickness distributions, respectively.

It should be noted that this model can be obtained directly from the two-media volume-averaged energy equations, as given in Vafai and Sozen [1990], by applying the one-dimensional assumptions, neglecting stream-wise thermal dispersion, and averaging the fluid-phase energy equation in the stream-wise direction. Equation (11) is the “fin equation”. Solutions analogous to conduction in common fin configurations, subject to appropriate boundary conditions, are easy to obtain. More complex configurations involving variable thickness or properties require a numerical solution.

The equivalent unit conductance of the porous wall is defined as

$$U_p = \frac{q(0)}{A(0)\left[T(0) - T_i\right]} \tag{14}$$

This is the unit surface conductance that the unenhanced fluid-solid interface would need in order to provide equal convective performance at the same coolant flow rate. In non-dimensional form it is given as

$$\hat{U}_p = \frac{U_p H}{k_e(0)} = -\Theta'(0) \tag{15}$$

Note that $\hat{U}_p k_e = \hat{k} \cdot \hat{N}u$ where $k_e$ is the coolant thermal conductivity and $\hat{N}u$ is the equivalent Nusselt number of the surface. Furthermore, in analogy to compact heat exchanger practice, we can define the porous matrix heat exchange effectiveness as

$$\eta_p = \frac{q(0)}{c_p G_o H W \left[T(0) - T_i\right]} = -\Theta'(0) \left(\frac{\eta_p}{\eta_m}\right) \tag{16}$$

where

$$\bar{m}H = \sqrt{\frac{c_p G_o H^2}{k_e(0)t(0)}} \tag{17}$$

In Eq. (16), the quantity $c_p G_o H W$ is the coolant capacity rate [Kays and Crawford, 1993].

**Porous Wall With Uniform Thickness and Composition**
Assume the porous wall has uniform thickness and composition. Then $k = 1$ and the coolant mass velocity is constant and equal to $G_0$. As a consequence, the coefficient $(mHF)^2$ is independent of $y$ and the solution to Eq. (11) for a porous wall having an insulated tip is

$$\Theta = \frac{\cosh[mHF(1 - \hat{y})]}{\cosh[mHF]}$$  

(18)

The non-dimensional equivalent unit conductance is

$$\hat{U}_p = mHF \tanh[mHF]$$

(19)

and the exchange matrix effectiveness becomes

$$\eta_p = \frac{F}{mH} \tanh[mHF]$$

(20)

Figure 4 shows the porous wall exchange effectiveness as a function of dimensionless mass velocity, $(mH)^2$ for three values of ntu. The effectiveness is maximum when ntu ≥ 2 and it approaches unity when $mH \rightarrow 0$. Thus, the exchange effectiveness is maximum when the porous wall operates at the thermodynamic limit (T = $T_0$), and the magnitude of $\eta_p$ decreases as the coolant mass velocity increases. This occurs because, at fixed ntu, an increase in mH causes a decrease in the temperature of the porous matrix. The effect is shown in Fig. 5, which is a plot of the non-dimensional temperature distribution, $\Theta(y)$, for $(mH) = 0.5, 1.0$ and $2.0$ with ntu = 2.

Figure 6 shows the non-dimensional equivalent unit surface conductance, $\hat{U}_p$, plotted versus coolant mass velocity, $(mH)^2$, with ntu = 0.5, 1.0, and ≥ 2.0. The figure shows that the equivalent conductance increases with coolant flow rate, and it is maximum when ntu ≥ 2.

The above results show that the cooling capacity of the porous wall is maximized when ntu ≥ 2. Assume the porous wall is composed of spherical particles of diameter, d. Particle Stanton number correlations show that $St = St(Re, Pr, \varepsilon)$. Furthermore, Eq. (1) shows that the product $\beta t = \beta(\varepsilon, t/d)$. Therefore, for a porous wall fabricated from spherical particles, ntu is a function of Re, Pr, $\varepsilon$ and t/d. Then, the requirement that ntu = 2 provides an equation for determining the optimal wall thickness-to-particle diameter ratio, $(t/d)_{opt}$, to maximize the
cooling capacity of the porous matrix for given particle Reynolds number, coolant Prandtl number and matrix porosity, $\varepsilon$;

$$ntu(Re, Pr, \varepsilon, (t/d)_{opt}) - 2 = 0 \quad (21)$$

The explicit form of Eq. (21) will depend on the Stanton number correlation employed to describe the particle-level convection process. Figure 7 plots $(t/d)_{opt}$ for $20 < Re < 10^3$. The Stanton number correlation of Wakao and Kaguei [1982] is used in structuring Eq. (21), and the porous wall is assumed to have a porosity of 40%. Results for three Prandtl numbers are shown, $Pr = 0.72, 5.8,$ and $50$. The optimal thickness-to-diameter ratio is seen to increase with both particle Reynolds number and coolant Prandtl number. If the coolant is air ($Pr = 0.72$), relatively thin walls (or large particles) are called for with $5 < (t/d)_{opt} < 10$. Thicker walls or smaller particles are favored when the coolant Prandtl number is large. For example, if the coolant is water, $5 < (t/d)_{opt} < 30$.

Assume the porous wall has height-to-thickness ratio $H/t = 5$, and it is fabricated from spherical copper particles resulting in $\varepsilon = 0.4$. The coolant is water at 300°K ($Pr = 5.8$). The porous matrix effective thermal conductivity is estimated as $k = 201$ watt/m°K [Duncan et al., 1989]. Figure 8 shows the resulting effective conductance, $\hat{U}_p$, as a function of coolant Reynolds number, $Re_H = \bar{G}_H H / \mu$. The coolant Reynolds number is related to the particle Reynolds number as follows

$$Re_H = \frac{t}{d} Re$$

In the figure, the maximum conductance, $\hat{U}_p[(t/d)_{opt}]$, is compared to the conductance obtained when the thickness-to-diameter ratio is held fixed at $t/d = 5, 10,$ and $15$, respectively, for all coolant flow rates. The figure shows that for a given coolant flow rate ($Re_H$), the maximum conductance is obtained when $t/d \geq (t/d)_{opt}$. This corresponds to operation at the thermodynamic limit ($ntu \geq 2$). As the coolant flow rate increases, the particle Stanton number, and hence $ntu$ decreases moving the operating condition into the convection-limited regime ($ntu < 2$). As a consequence, $\hat{U}_p$ drops below the maximum value. This situation is depicted by the three dashed trajectories shown in the Figure.

It is interesting to note that while the conductance is maximized when $ntu \geq 2$, the coolant pressure drop is minimized when $ntu \leq 2$. Figure 9 plots the porous matrix friction coefficient, $C_f$, versus $Re_H$ for the same conditions as Fig. 8. The solid line shows the friction coefficient obtained when $t/d = (t/d)_{opt}$ at each coolant flow rate, and the broken lines show the situation when $t/d$ is held fixed while $Re_H$ varies. When $t/d \geq$...
(t/d)$_{\text{opt}}$, $C_f$ is greater than that obtained with $t/d = (t/d)_{\text{opt}}$ for a given coolant flow rate. The friction coefficient drops below that obtained with the optimal thickness-to-diameter ratio when the flow rate increases into the convection-limited operating regime.

For a given coolant flow rate, both the conductance and the pressure drop (and coolant pumping power) are optimized when the porous exchange matrix is designed to operate at $\text{ntu} = 2$.

**Overall System Design**

Assume the porous wall is bonded to a base plate at temperature, $T_b$, and the bond thermal resistance per unit base plate area is $R_i''$ [°K m$^2$/watt]. Then the porous media temperature distribution is related to $T_b$ as follows:

$$\frac{T - T_i}{T_b - T_i} = \frac{1}{1 + U_p R_i''} \Theta(\hat{y})$$

Furthermore, in addition to conduction/convection through the porous matrix, heat is directly convected from the base plate to the coolant from exposed surfaces within the base plate - porous matrix interface. The total heat transfer rate, $q_{\text{tot}}$ is given as

$$q_{\text{tot}} = \frac{U_p A(0)[T_b - T_i] + h_i \varepsilon A(0) \left[ T_b - \frac{T_i(0) + T_e}{2} \right]}{1 + U_p R_i''}$$

In eq. (23), $h_i$ is the unit conductance of exposed surfaces within the base plate-porous matrix interface. The magnitude of $h_i$ depends on local conditions within the porous matrix near the interface. Assume $T_i(0) \sim T_e$ and $h_i \sim h_p$ (the particle heat transfer coefficient). Then, eq. (23) can be rewritten in terms of the overall non-dimensional unit conductance, $\hat{U}_{\text{tot}}$, as

$$\hat{U}_{\text{tot}} \equiv \frac{q_{\text{tot}} A H}{k_c(0)A(0)[T_b - T_i]} = \frac{\hat{U}_p}{1 + U_p R_i''} + \frac{\varepsilon}{2} S_t \frac{\text{Re Pr}}{\text{Re H}}$$

It is noted that the second term on the right side of Eq. (24) provides a 25% - 30% increase in the magnitude of the effective conductance. Furthermore, if it is assumed that the interface resistance is 25% of the porous wall thermal resistance, $R''_i = 0.25 U_p^{-1}$, then $U_{\text{tot}}$ is reduced by approximately 15%.
Conclusions

A semi-empirical model for conduction/convective conduction in a thin “fin-like” porous wall of arbitrary composition and thickness is developed. The model assumes one-dimensional conduction in the porous matrix and one-dimensional flow of the coolant through the wall. Conduction within the porous matrix is coupled to the coolant using Newton’s cooling law together with appropriate correlations for the particle heat transfer coefficient. The coolant flow distribution within the porous matrix is established using an empirically based pressure drop correlation.

The model admits to two cooling modes: a convection-limit and a thermodynamic-limit. The convection limit occurs when the temperature increase of the coolant is limited by Newton’s rate equation. This corresponds to coolant flow where the local number of transfer units of the porous matrix is less than two (ntu < 2). The thermodynamic limit occurs when the coolant temperature equals the porous media temperature as it exits the media. This occurs when ntu ≥ 2.

Closed form solutions are obtained for a uniform composition and thickness porous wall having an insulated tip. The coolant mass velocity distribution is uniform under these conditions. Analysis shows that the equivalent unit surface conductance and the effectiveness of the porous wall is always maximum when it is operated with ntu ≥ 2. Furthermore, if the porous matrix is composed of a packed bed of spherical particles, it is found that, for a given coolant flow rate, the pressure drop across the porous matrix is minimum when $t / d = t / d_{\text{opt}}$ ntu ≤ 2. This suggest that for many design requirements, the porous media exchanger should be operated at ntu = 2 with $t / d = t / d_{\text{opt}}$.

Equations which integrate the current porous media heat transfer model into an overall system concept are developed. It is found that performance is maintained by insuring that the bond between the porous matrix and its supporting base material is “clean”, with minimum interfacial thermal resistance.

References


