2. Since $f$ is never zero, there are two cases: either $f(x) > 0$ for all $x \in I$ or $f(x) < 0$ for all $x \in I$. In the first case, take $m = \inf_I f > 0$ and then $f(x) \geq m$ for all $x$. In the second case, take $m = -\sup_I f > 0$ since $\sup_I f < 0$ and then $f(x) \leq -m$ for all $x$.

3. The points on the graph are $(x, f(x))$ with $x \in [a, b]$. The distance to $(x_0, y_0)$ is $d(x) = \sqrt{(x-x_0)^2 + (f(x)-y_0)^2}$. Since $d : [a, b] \to [0, \infty)$ is a continuous function, it has a minimum $d(x_m)$. Then $(x_m, f(x_m))$ is closest to $(x_0, y_0)$.

4. Let $f : (0, 1] \to \mathbb{R}, f(x) = \frac{1}{x}$. Then $f$ is continuous and unbounded.

Let $g : (-1, 1) \to \mathbb{R}, g(x) = 2x - 1$. Then $g$ is continuous and bounded, $-3 \leq g(x) \leq 1$, but it does not have a maximum (or a minimum).

5. Let $f : [1, \infty) \to \mathbb{R}, f(x) = x^2$. Then $f$ is continuous and unbounded.

Let $g : [1, \infty) \to \mathbb{R}, g(x) = -\frac{1}{x}$. Then $g$ is continuous and $-1 \leq g(x) \leq 0$, but it does not have a maximum.

8. Consider $h : [a, b] \to \mathbb{R}, h(x) = f(x) - g(x)$. Then $h$ is continuous and $h(a) < 0, h(b) > 0$. By the Intermediate Value Theorem, there is $c \in (a, b)$ such that $h(c) = 0$, i.e. $f(c) = g(c)$.

10. Consider $f : [0, \infty) \to [0, \infty), f(x) = x^n$ which is continuous. Given $a \in [0, \infty)$, since $k^n \to \infty$ when $k \to \infty$, we can find $N \in \mathbb{N}$ with $N^n \geq a$. By the Intermediate Value Theorem, there is $x \in [0, N]$ with $x^n = a$ or $x = \sqrt[n]{a}$.

12. If $f(b) > m$, for any $y \in (m, f(b))$, since $f(a) \leq m < y < f(b)$ we would find $c \in (a, b)$ such that $f(c) = y > m$, contradiction with $f(c) \leq m$. Hence it must be that $f(b) \leq m$. 