1. Prove that if $\inf A < \infty$ and $n \in \mathbb{N}$, then there is $a_n \in A$ such that

$$\inf A \leq a_n < \inf A + \frac{1}{n}.$$ 

Since $\inf A$ is the greatest lower bound, $\inf A + \frac{1}{n}$ is no longer a lower bound, so there is $a_n \in A$ with $a_n < \inf A + \frac{1}{n}$. Obviously $\inf A \leq a_n$.

2. Use the definition of limit to prove that $\lim \sqrt{\frac{n}{n+1}} = 1$.

$$\left| \sqrt{\frac{n}{n+1}} - 1 \right| = \left| \frac{(\sqrt{\frac{n}{n+1}} - 1)(\sqrt{\frac{n}{n+1}} + 1)}{\sqrt{\frac{n}{n+1}} + 1} \right| =$$

$$\left| \frac{n}{n+1} - 1 \right| < \frac{1}{n+1}$$

Given $\varepsilon > 0$, let $N = \frac{1}{\varepsilon} - 1$. Then for $n > N$ we have $\frac{1}{n+1} < \varepsilon$, so $\left| \sqrt{\frac{n}{n+1}} - 1 \right| < \varepsilon$. 
