QUIZ No. 5, Math 311 Fall 2008

NAME SOLUTIONS

October 3, 2008

1. A set $E \subset \mathbb{R}^n$ is sequentially compact if every sequence in E has a convergent subsequence whose limit belongs to E. Prove that any compact set in $\mathbb{R}^n$ is sequentially compact.

Let E be compact. Then E is closed and bounded

Let $\{x_k\} \subset E$. Since E is bounded, B-W theorem implies there is a convergent subsequence $\{x_{k_j}\}$, let $x_{k_j} \to x$. Since E is closed, $x \in E$

Hence E is sequentially compact

2. Prove that $\lim_{(x,y)\to(0,0)} \dfrac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} = 1$.

Let $x = r\cos\theta \quad y = r\sin\theta$

$$\lim_{(x,y)\to(0,0)} \frac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} = \lim_{r\to 0} \frac{\sin r}{r} = 1$$