1. Use the definition \[ \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st}f(t)dt \] to find the Laplace transform of
\[ f(t) = \begin{cases} -t, & 0 \leq t < 1 \\ 2t - 2, & t \geq 1 \end{cases}. \]
Then express \( f(t) \) using Heaviside functions, and check your answer.

2. Find the general solution of \( x^2y'' + xy' - y = 2x \). Recall
\[ y_p = u_1y_1 + u_2y_2, \quad u_1' = -\frac{y_2f(x)}{W(y_1,y_2)}, \quad u_2' = \frac{y_1f(x)}{W(y_1,y_2)}. \]

3. Find two linearly independent solutions of
\[ X' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} X. \]
Compute their Wronskian.

4. Solve the IVP \( y'' + y = f(t), y(0) = 0, y'(0) = 1 \), where
\[ f(t) = \begin{cases} \cos t, & 0 \leq t < \pi \\ -\sin t, & t \geq \pi \end{cases}. \]

5. Find the general solution of the nonhomogeneous differential equation
\[ y'' + 4y' + 4y = t^2. \]
using the method of undetermined coefficients, and then by using the Laplace transform, assuming \( y(0) = a, y'(0) = b \).

6. A spring-mass system has a spring constant \( 3N/m \). A mass of \( 2kg \) is attached to the spring and the motion takes place in a viscous fluid that offers a resistance numerically equal to the magnitude of the instantaneous velocity. If the system is driven by an external force of \( 3\cos 3t - 2\sin 3t \ N \), determine the steady state response.

7. Solve the initial value problem \( y'' - 2y' + 2y = e^x \tan x, y(0) = 1, y'(0) = 1/2 \) using variation of parameters.

8. Solve the initial value problem \( x^2y'' + xy' + y = 0, y(1) = 1, y'(1) = 2 \).

9. Air containing 0.06% carbon dioxide is pumped into a room whose volume is 8000 ft\(^3\). The air is pumped in at a rate of 2000 ft\(^3\)/min, and the circulated air is then pumped out at the same rate. If there is an initial concentration of 0.2% carbon dioxide in the room, determine the subsequent amount in the room after 10 minutes.
10. Solve the IVP \((\sin x)y' + (\cos x)y = 0, y(7\pi/6) = -2\), and find the largest interval \(I\) on which the solution is defined.

11. An LR series circuit has a variable inductor with the inductance defined by

\[
L(t) = \begin{cases} 
1 - \frac{t}{10}, & 0 \leq t < 10 \\
0, & t \geq 10 
\end{cases}
\]

Find the current \(i(t)\) if the resistance is 0.2 ohm, the impressed voltage is \(E(t) = 4\), and \(i(0) = 0\).

12. A cylindrical tank 10 feet high has a radius of 2 feet and it is leaking water through a circular hole of radius 1/2 inch. Using the equation

\[
\frac{dh}{dt} = -\frac{Ah}{Aw} \sqrt{2gh}, h(0) = 10
\]

where \(g = 32 \text{ ft/s}^2\), how long will it take to empty?

13. Find the inverse Laplace transform of \(\frac{(1 + e^{-2s})^2}{(s + 2)^2}\).

14. Consider the vibrating system described by \(u'' + u = 3 \cos \omega t, u(0) = 1, u'(0) = 1\). Find the solution for \(\omega \neq 1\).

15. Find the solution of the initial value problem \(y'' + 4y = U(t - \pi) - U(t - 3\pi); y(0) = 0, y'(0) = 1\).

16. Find the solution of the initial value problem

\[
y'' + 2y' + 2y = 2\delta(t - \pi) - \delta(t - \pi/2), \ y(0) = 0, \ y'(0) = 0.
\]

17. Solve the system of equations

\[
x_1' = -0.5x_1 + 2x_2, \ x_1(0) = 1
\]

\[
x_2' = -2x_1 - 0.5x_2, \ x_2(0) = 2.
\]

18. Use the Laplace transform to solve

\[
y' + 6y + 9\int_0^t y(\tau)d\tau = 1, y(0) = 1.
\]

19. Find the general solution of the system

\[
X' = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 6 & 0 \\ -4 & 0 & 4 \end{pmatrix} X.
\]

20. Prove that the general solution of \(X' = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} X + \begin{pmatrix} -5 \\ 3 \end{pmatrix}\) is

\[
X(t) = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ -3 \end{pmatrix} e^{-t} + \begin{pmatrix} 1 \\ 3 \end{pmatrix}.
\]