REVIEW FOR TEST No. 1, Math 283 Spring 2016

1. Identify the quadric surfaces

\[ x^2 + 2z^2 = 8, \quad x = y^2 - z^2, \quad x^2 - 2x + y^2 + z^2 = 0. \]

2. Find parametric equations and symmetric equations for the line obtained by intersecting the planes \( x - y + 2z = 1 \) and \( 2x + y - z = 4 \).

3. Find and sketch the domain of \( z = f(x, y) = \ln(y^2 - x - y) \). Draw the level curves corresponding to levels \( z = -1, z = 0, z = 1 \).

4. Find the volume of the parallelipiped determined by the vectors \( \mathbf{i} + \mathbf{j} + \mathbf{k}, \mathbf{i} + \mathbf{j} + \mathbf{k} \).

5. Consider the points \( P(1, -1, 2), Q(2, 1, 4) \) and \( R(2, 0, 3) \). Find the area of the triangle \( PQR \) and the equation of the plane containing \( P, Q, R \).

6. Given the curve with parametric equations \( x = \sin^2 t, y = \sin t, \pi/2 \leq t \leq \pi \), find a Cartesian equation by eliminating the parameter and then sketch the curve, indicating how it is traced.

7. Let \( \mathbf{r}(t) = \langle 3\cos 2t, -3\sin 2t, 5t \rangle \). Find the unit normal vector \( \mathbf{N} \) and the curvature \( \kappa = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} \) at \( t = \pi/2 \).

8. Find an equation of the plane containing both the point \( (1, 2, -3) \) and the line with parametric equations \( x = 2 - 3t, y = 1 + 2t, z = -t \).

9. A particle of mass 3 starts at the point \( (0, -1, 1) \) with initial velocity \( \mathbf{i} - \mathbf{j} + 2\mathbf{k} \) and it is moving under the influence of the force \( \mathbf{F} = 3e^t \mathbf{i} + \mathbf{j} - \cos t \mathbf{k} \). Find its position function at any time.

10. Find the tangential and normal components of the acceleration vector of a particle with position function \( \mathbf{r}(t) = t\mathbf{i} + 2e^t \mathbf{j} + \cos t \mathbf{k} \) at \( t = 0 \). Recall:

\[
\begin{align*}
    a_T &= \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{|\mathbf{r}'(t)|}, \\
    a_N &= \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}.
\end{align*}
\]
11. Find all points where the following functions are continuous

\[ f(x, y) = \ln |xy|, \quad g(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x, y) \neq (0, 0) \\ 1, & (x, y) = (0, 0). \end{cases} \]

12. Find \( f_x, f_y, f_{xx} \) and \( f_{xy} \) if

\[ f(x, y) = x^2e^{xy}, \quad f(x, y) = x \sin(xy). \]

13. Compute the limits (if they exist)

\[ \lim_{{(x, y) \to (0, 0)}} xy \ln(x^2+y^2), \quad \lim_{{(x, y) \to (2, 1)}} \frac{e^{x^2-4y} - 1}{x^2 - 4y}. \]

14. Find \( \partial z/\partial x \) at the point \((0, 1, -1)\) if the equation \( x^2y + z^2x - 2z = 2 \) defines \( z \) as a function of \( x, y \).

15. Use the chain rule to find \( \partial w/\partial r \) and \( \partial w/\partial s \) when \( r = 1, s = -1 \) if

\[ w = \ln(x + y + z), \quad x = r^2 - s^2, \quad y = \cos(r + s), \quad z = \sin(r + s). \]

16. Find the linearization of \( f(x, y) = e^x \cos y \) at \((0, \pi/2)\) and the equation of the tangent plane at \((0, \pi/2, 0)\).