1. A rectangular box without a lid is to have a volume of 32,000 cm$^3$. Find the dimensions that minimize the amount of cardboard used.

2. Find the directional derivative of $f(x, y) = \ln(2x^2 - y^2)$ at $(-1, 1)$ in the direction of $3\mathbf{i} - 4\mathbf{j}$.

3. Evaluate $\int \int_E xydV$ where $E$ is bounded by the parabolic cylinders $y = x^2$ and $x = y^2$ and the planes $z = 0$ and $z = x + y$.

4. Find equations of the tangent plane and the normal line at $(1, -1, 3)$ on the surface $x^2 + 2xy - y^2 + z^2 = 7$.

5. Find and classify the critical points of $f(x, y) = x^3 - 6xy + 8y^3$.

6. Find the area enclosed by one leaf of the rose $r = 4 \cos 3\theta$.

7. Rewrite the integral $\int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} f(x, y, z)dzdydx$ as an iterated integral in the order $dxdydz$ and $dxdzdy$.

8. Find the extreme values of $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ in the elliptic region $x^2 + 2y^2 \leq 16$.

9. Reverse the order of integration and then compute $\int_{0}^{1} \int_{x^2}^{1} x^3 \sin(y^3)dydx$.

10. Find the volume of the solid bounded by the surface $z = \sqrt{4 - x^2 - y^2}$ and the planes $z = 0, y = 0, y = x$ in the first octant.

11. Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the plane $z = 0$ and inside the cylinder $x^2 + y^2 = 2x$.

12. Find the average value of $f(x, y) = \frac{y}{1 + x^2y^2}$ over $R = [0, 1] \times [0, 4]$.

13. Find the points on the surface $z^2 = xy + 1$ that are closest to the point $(1, 0, 0)$.

14. Identify the curve $r = 2 \cos \theta$ by finding a Cartesian equation.

15. A lamina occupies the part of the disc $x^2 + y^2 \leq 4$ that lies in the fourth quadrant. Find the center of mass of the lamina if the density function is $\delta(x, y) = -xy$.

16. Find the area inside $r = 1 + \cos \theta$ and outside $r = \cos \theta$. 