Newton-Raphson Method

Lecture Plan

1. Modified Pig Problem
   a. Formulation
   b. Graphical Solution
   c. Maple Solution
   d. Equations that Cannot Be Solved Algebraically

2. Newton-Raphson Method
   a. Method and Maple implementation
   b. Sensitivity using the Newton-Raphson method

3. solve and fsolve commands in Maple

Home Work: Chapter 3, Sect. 3.5, Ex. 1 (due on Thursday, Feb. 19)
• Recall the pig problem of Lecture 2:

**Example 1.1.** A pig weighing 200 pounds gains 5 pounds per day and costs 45 cents a day to keep. The market price for pigs is 65 cents per pound, but is falling 1 cent per day. When should the pig be sold?

**Variables:**
- \( t \) = time (days)
- \( w \) = weight of pig (lbs)
- \( p \) = price for pigs (\$/lb)
- \( C \) = cost of keeping pig \( t \) days (\$)
- \( R \) = revenue obtained by selling pig (\$)
- \( P \) = profit from sale of pig (\$)

**Assumptions:**
- \( w = 200 + 5t \)
- \( p = 0.65 - 0.01t \)
- \( C = 0.45t \)
- \( R = p \cdot w \)
- \( P = R - C \)
- \( t \geq 0 \)

**Objective:** Maximize \( P \)

• Today we assume that the pig growth rate is not constant. It changes as the pig is growing. More specifically, we assume that the growth rate is proportional to the pig’s weight (see Maple code for further details).
Newton-Raphson Method

Sir Isaac Newton (25 December 1642 – 20 March 1726)

An English physicist and mathematician (described in his own day as a "natural philosopher") who is widely recognized as one of the most influential scientists of all time and as a key figure in the scientific revolution. His book Philosophiæ Naturalis Principia Mathematica ("Mathematical Principles of Natural Philosophy"), first published in 1687, laid the foundations for classical mechanics. Newton also made seminal contributions to optics and shares credit with Gottfried Leibniz for the invention of calculus.

Joseph Raphson was an English mathematician known best for the Newton–Raphson method. Little is known about his life, and even his exact years of birth and death are unknown, although the mathematical historian Florian Cajori provided the approximate dates 1648–1715.

Raphson's most notable work is Analysis Aequationum Universalis, which was published in 1690. It contains a method, now known as the Newton–Raphson method, for approximating the roots of an equation. Isaac Newton had developed a very similar formula in his Method of Fluxions, written in 1671, but this work would not be published until 1736, nearly 50 years after Raphson's Analysis. However, Raphson's version of the method is simpler than Newton's, and is therefore generally considered superior. For this reason, it is Raphson's version of the method, rather than Newton's, that is to be found in textbooks today.
The **Newton–Raphson method** is a method for finding successively better approximations to the roots (or zeroes) of a real-valued function

\[ x : f(x) = 0. \]

The Newton–Raphson method in one variable is implemented as follows:

Given a function \( f \) defined over the reals \( x \), and its derivative \( f' \), we begin with a first guess \( x_0 \) for a root of the function \( f \). Provided the function satisfies all the assumptions made in the derivation of the formula, a better approximation \( x_1 \) is

\[ x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}. \]

Geometrically, \((x_1, 0)\) is the intersection with the \( x \)-axis of the tangent to the graph of \( f \) at \((x_0, f(x_0))\).

The process is repeated as

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

until a sufficiently accurate value is reached.