Solution for Section 2.3

Due Feb 15, 07

2. c) \( P(\text{more then 3 fuses selected}) = P(\text{first three are 10A}) \)
\[
= \left( \frac{8}{10} \times \frac{7}{9} \times \frac{6}{8} \right) = \frac{7}{15}
\]

4. \((0.056)(0.027) = 0.001512.\)

6. Let \(A\) denote the event that the allocation sector is damaged, and let \(N\) denote the event that a non-allocation sector is damaged.

Then, \(P(A \cap N^c) = 0.20, P(A^c \cap N) = 0.70, \) and \(P(A \cap N) = 0.10\)

a) \(P(A) = P(A \cap N^c) + P(A \cap N) = 0.30\)

b) \(P(N) = P(A^c \cap N) + P(A \cap N) = 0.80\)

c) \(\frac{P(N \cap A)}{P(A)} = \frac{P(A \cap N)}{P(A \cap N) + P(A \cap N^c)} = \frac{0.10}{0.10 + 0.20} = \frac{1}{3}\)

14. a) \(P(A) = \frac{300}{1000} = \frac{3}{10} = 0.3\)

b) Given that \(A\) occurs, there are 999 components remaining, of which 299 are defective. Therefore, \(P(B \mid A) = \frac{299}{999}\)

c) \(P(A \cap B) = P(A)P(B \mid A) = \frac{1}{3} \times \frac{299}{999} = \frac{299}{3330}\)

d) Given that \(A^c\) occurs, there are 999 components remaining, of which 300 are defective. Therefore, \(P(B \mid A) = \frac{300}{999}.\)

Now, \(P(A^c \cap B) = P(A^c)P(B \mid A^c) = \frac{7}{10} \times \frac{300}{999} = \frac{70}{333}\)

e) \(P(B) = P(A \cap B) + P(A^c \cap B) = \frac{299}{3330} + \frac{70}{333} = \frac{3}{10} = 0.3\)

f) \(P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{299}{3330} \times \frac{10}{3} = \frac{299}{999}\)

g) A and B are not independent, but they are very nearly independent. To see this that \(P(B) = 0.3,\) while \(P(B \mid A) = 0.2993.\) So, \(P(B)\) is very nearly equal to \(P(B \mid A),\) but not exactly equal. Alternatively, note that \(P(A \cap B) = 0.0898,\) while \(P(A)P(B) = 0.09.\) Therefore, in most situations it would be reasonable to treat A and B as though they were independent.
18. Let Fl denote the event that a bottle has a flow. Let F denote the event that a bottle fails inspection. We are given \( P(Fl) = 0.0002, P(F/Fl) = 0.995 \), and 
\[ P(F^c / Fl^c) = 0.99 \]

\[ P(\text{Fl} / F) = \frac{P(F / Fl)P(Fl)}{P(F / Fl)P(Fl) + P(F / Fl^c)P(Fl^c)} \]
\[ = \frac{P(F / Fl)P(Fl)}{P(F / Fl)P(Fl) + [1 - P(F^c / Fl^c)]P(Fl^c)} \]
\[ = \frac{(0.995)(0.0002)}{(0.995)(0.0002) + (1 - 0.99)(0.9998)} = 0.01952 \]

b) i. Given that a bottle failed inspection, the probability that it had a flaw is only 0.01952.

c) \[ P(\text{Fl}^c / F^c) = \frac{P(F^c / Fl^c)P(Fl^c)}{P(F^c / Fl^c)P(Fl^c) + P(F^c / Fl)P(Fl)} \]
\[ = \frac{P(F^c / Fl^c)P(Fl^c)}{P(F^c / Fl^c)P(Fl^c) + [1 - P(F / Fl)]P(Fl)} \]
\[ = \frac{(0.99)(0.9998)}{(0.99)(0.9998) + (1 - 0.995)(0.0002)} = 0.999999 \]

d) ii. Given that a bottle passes inspection, the probability that is has no flaw is 0.999999.

e) The small probability in part a) indicates that some good bottles will be scrapped. This is not so serious. The important thing is that of the bottles that pass inspection, very few should have flaws. The large probability in part c) indicates that is the case.

22. \( P(\text{system functions}) = P[(A \cap B) \cup (C \cup D)] \).

Now, \( P(A \cap B) = P(A)P(B) = (1 - 0.1)(1 - 0.05) = 0.855 \). And
\( P(C \cup D) = P(C) + P(D) - P(C \cap D) = (1 - 0.1) + (1 - 0.2) - (1 - 0.1)(1 - 0.2) = 0.98 \).

Therefore,
\[ P[(A \cap B) \cup (C \cup D)] = P(A \cap B) + P(C \cup D) - P[(A \cap B) \cap (C \cup D)] \]
\[ = P(A \cap B) + P(C \cup D) - (A \cap B)P(C \cup D) \]
\[ = 0.855 + 0.98 - (0.855)(0.98) \]
\[ = 0.9971 \]
Solution for Section 2.4

Due Feb 15, 07

2. a) \( P(X < 2) = P(X = 0) + P(X = 1) = p(0) + p(1) = 0.5 + 0.3 = 0.8 \)

b) \( P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - p(0) = 1 - 0.5 = 0.5 \)

c) \( \mu_X = 0(0.5) + 1(0.3) + 2(0.1) + 3(0.1) = 0.8 \)

d) \( \sigma_X^2 = (0 - 0.8)^2(0.5) + (1 - 0.8)^2(0.3) + (2 - 0.8)^2(0.1) + (3 - 0.8)^2(0.1) = 0.96 \)
   Alternatively, \( \sigma_X^2 = (0)^2(0.5) + (1)^2(0.3) + (2)^2(0.1) + (3)^2(0.1) - (0.8)^2 = 0.96 \)

4. a) \( p_x(x) \) is the only probability mass function, because it is the only one whose probability sum to 1.

b) \( \mu_X = 0(0.1) + 1(0.2) + 2(0.4) + 3(0.2) + 4(0.1) = 2 \)
   \( \sigma_X^2 = (0 - 2)^2(0.1) + (1 - 2)^2(0.2) + (2 - 2)^2(0.4) + (3 - 2)^2(0.2) + (4 - 2)^2(0.1) = 1.2 \)

8. Let \( A \) denote an acceptable chip, and \( U \) an unacceptable one.

a) \( P(A) = 0.9 \)

b) \( P(UA) = P(U)P(A) = (0.1)(0.9) = 0.09 \)

c) \( P(X = 3) = P(UUA) = P(U)P(U)P(A) = (0.1)(0.1)(0.9) = 0.009 \)

d) \( p(x) = \begin{cases} (0.9)(0.1)^{x-1}, & x = 1, 2, 3, \ldots \\ 0, & otherwise \end{cases} \)

10. Let \( S \) denote Success occur and \( F \) denote Failure occurs.

a) 0, 1, 2, 3

b) \( P(X = 3) = P(SSS) = (0.8)^3 = 0.512 \)

c) \( P(FSS) = (0.2)(0.8)^2 = 0.128 \)

d) \( P(SFS) = P(SSF) = (0.8)^2(0.2) = 0.128 \)

e) \( P(X = 2) = P(FSS) + P(SFS) + P(SSF) = 3(0.8)^2(0.2) = 0.384 \)
f) \[ P(X = 1) = P(FFS) + P(FSF) + P(SFF) = 3(0.8)(0.2)^2 = 0.096 \]

g) \[ P(X = 0) = P(FFF) = (0.2)^3 = 0.008 \]

h) \[ \mu_X = 0(0.08) + 1(0.096) + 2(0.384) + 3(0.512) = 2.4 \]

i) \[ \sigma^2_X = (0 - 2.4)^2(0.08) + (1 - 2.4)^2(0.096) + (2 - 2.4)^2(0.384) + (3 - 2.4)^2(0.512) \]
\[ = 0.89472 \]

j) \[ P(Y = 3) = P(FSSS) + P(SFSS) + P(SSFS) + P(SSSF) \]
\[ = 4(0.8)^3(0.2) = 0.4096 \]