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1. In a group of college students, 30% have Visa credit card, 20% have Master credit card and 5% have cards of both types. A student is selected at random.

   a) What is the probability that the selected student have either Visa or Master card?

   b) If the selected student has a Master card, what is the probability that he also have a Visa card?

   c) Are events "student has Visa card” and "student has Master card” independent?
2. A researcher eliminated one outlier \( x = 100 \) from a sample. This changed the sample mean from \( m = 1 \) to \( m = 0 \). How many observations were in the original sample?

3. A metal-weapon detector gives an alarm in 99\% of cases when an armed person is scanned. The same detector will give a false alarm only in 5\% of cases when an unarmed person is scanned. The probability that a scanned person actually carries a weapon is 0.001. If detector gives an alarm, what is the probability that the scanned person actually has a weapon?
4. A shuttle is coming to a bus stop sometime between 9:00 and 9:10AM. Mary is going to catch this shuttle; and she is coming to the bus stop during the same time interval. Assume that the shuttle arrival time $Y$ and Mary’s arrival time $X$ are independent and uniformly distributed within [9:00, 9:10]AM (that is, the pdfs $f_X(x)$ and $f_Y(y)$ are constants within this interval and zero otherwise).

a) Find the joint pdf $f(x, y)$ of $X$ and $Y$?

b) What is the probability that Mary will catch the shuttle (i.e. that $X \leq Y$)?

[Hint: Use graphical representation of the two random variables, find the region that corresponds to $X \leq Y$, calculate the appropriate integral over this region.]