Problem 1
The true relationship between random variables $X$ and $Y$ is given by the linear regression equation:

$$Y = 5 - 3X + \varepsilon,$$

where $\varepsilon \sim N(0, 3^2)$. The regression was estimated using a data set $(x_i, y_i)$ as

$$\hat{Y} = 4 - 2.5X.$$

a) Write the equation of the true regression line.
b) Sketch the true regression line and the estimated regression line in the same axes.
c) Find the value of $Y$ that corresponds to $X = 1$ on the true regression line.
d) Find the conditional distribution of $Y$ given that $X = 2$.
e) Find the conditional mean of $Y$ given that $X = 2$.
f) Find the conditional variance of $Y$ given that $X = 2$.
g) Compare the conditional mean and conditional variance of $Y$ given $X = 2.5$.
h) Does the conditional mean of $Y$ depend on the values of $X$?
i) Does the conditional variance of $Y$ depend on the values of $X$?
j) Find the distribution of a random variable $Z = Y + 3X$.
k) Find the conditional distribution of a variable $Z = Y + X$ given $X = 2$.

Next assignments are for a measurement $(X, Y) = (1, 1)$.

l) Add the measurement to the plot of b).
m) Find the measurement error.
n) Find the predicted (fitted) values of $Y$ according to the linear regression fit (2).
o) Find the residual.
p) Predict $Y$ at $X = 1, 2, 3, 4$.
q) If $x_1$ and $x_2$ differ by 3, by how much do we predict the corresponding $Y$ values to differ?
r) What should be the values of $X$ in order for the predicted value of $Y$ to be 0?
s) All measured values of $Y$ in a certain group are greater than 0. Can you conclude that all the values of $X$ in this group are less than the value found in r)?

Problem 2
Each month for several months, the average temperature $x$ in °C and the number $y$ of pounds of steam consumed by a certain chemical plant were measured. The fitted regression line has the form

$$y = 245.82 + 1.13x.$$

a) Predict the number of pounds of steam consumed in a month for the average temperature 20 °C.
b) It is known that the average temperatures in January and July differ by 10 °C. By how much do you predict the number of pounds of steam consumed to differ?
c) Will the predicted amount of steam consumed increase or decrease with the average temperature?
d) When do you think the amount of the steam consumed will be higher: in the winter or in the summer?

e) What should be the average temperature in order for the predicted value of the steam consumed to be 260?

f) Find the residual of a measurement \((x = 10, y = 260)\)

g) Can you find the measurement error in f)? Find the error value or explain why this can not be done.