Section 7.3

7.3.1

1. (a) \( \bar{x} = 65.0, \ y = 29.05, \ \Sigma_{i=1}^{n}(x_i - \bar{x})^2 = 6032.0, \ \Sigma_{i=1}^{n}(y_i - \bar{y})^2 = 835.42, \ \Sigma_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = 1988.4, \ n = 12. \)

\[ \hat{\beta}_1 = \frac{\Sigma_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\Sigma_{i=1}^{n}(x_i - \bar{x})^2} = 0.329642 \text{ and } \hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 7.623276. \]

(b) \( r^2 = \frac{\Sigma_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})^2}{\Sigma_{i=1}^{n}(x_i - \bar{x})^2 \Sigma_{i=1}^{n}(y_i - \bar{y})^2} = 0.784587. \)
\[ s^2 = \frac{(1 - r^2) \Sigma_{i=1}^{n}(y_i - \bar{y})^2}{n - 2} = 17.996003. \]

(c) \( s = \sqrt{17.996003} = 4.242170. \)
\[ s_{\hat{\beta}_0} = s \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{\Sigma_{i=1}^{n}(x_i - \bar{x})^2}} = 3.755613. \]
\[ s_{\hat{\beta}_1} = \frac{s}{\sqrt{\Sigma_{i=1}^{n}(x_i - \bar{x})^2}} = 0.0546207. \]

There are \( n - 2 = 10 \) degrees of freedom. \( t_{10,0.025} = 2.228. \)

Therefore a 95% confidence interval for \( \beta_0 \) is \( 7.623276 \pm 2.228(3.755613), \) or \((-0.744, 15.991). \)

The 95% confidence interval for \( \beta_1 \) is \( 0.329642 \pm 2.228(0.0546207), \) or \((0.208, 0.451). \)

(d) \( \hat{\beta}_1 = 0.329642, \ s_{\hat{\beta}_1} = 0.0546207, \ n = 12. \) There are \( 12 - 2 = 10 \) degrees of freedom.

The null and alternate hypotheses are \( H_0: \beta_1 \geq 0.5 \) versus \( H_1: \beta_1 < 0.5. \)
\[ t = (0.329642 - 0.5)/0.0546207 = -3.119. \]

Since the alternate hypothesis is of the form \( \beta_1 < b, \) the \( P \)-value is the area to the left of \( t = -3.119. \)

From the \( t \) table, \( 0.005 < P < 0.01. \) A computer package gives \( P = 0.00545. \)

We can conclude that the claim is false.

(e) \( x = 40, \ \bar{y} = 7.623276 + 0.329642(40) = 20.808952. \)
\[ s_{\bar{y}} = s \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\Sigma_{i=1}^{n}(x_i - \bar{x})^2}} = 1.834204. \] There are 10 degrees of freedom. \( t_{10,0.025} = 2.228. \)

Therefore a 95% confidence interval for the mean response is \( 20.808952 \pm 2.228(1.834204), \) or \((16.722, 24.896). \)

(f) \( x = 40, \ \bar{y} = 7.623276 + 0.329642(40) = 20.808952. \)
\[ s_{\text{pred}} = s \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{\Sigma_{i=1}^{n}(x_i - \bar{x})^2}} = 4.621721. \] There are 10 degrees of freedom. \( t_{10,0.025} = 2.228. \)

Therefore a 95% prediction interval is \( 20.808952 \pm 2.228(4.621721), \) or \((10.512, 31.106). \)
7.3.2

2. (a) \( n - 2 = 15 - 2 = 13 \)

(b) \( \hat{\beta}_1 = 2.9349, s_{\hat{\beta}_1} = 0.086738, t_{13,0.01} = 2.650, \) so a 98% confidence interval is \( 2.9349 \pm 2.650(0.086738), \) or \( (2.705, 3.165). \)

(c) \( \hat{\beta}_0 = 6.6361, s_{\hat{\beta}_0} = 1.1455, t_{13,0.01} = 2.650, \) so a 98% confidence interval is \( 6.6361 \pm 2.650(1.1455), \) or \( (3.601, 9.672). \)

(d) The null and alternate hypotheses are \( H_0: \beta_1 = 0.35 \) versus \( H_1: \beta_1 \neq 0.35. \)
\[ \hat{\beta}_1 = 2.9349, s_{\hat{\beta}_1} = 0.086738, t = (2.9349 - 0.35)/0.086738 = 29.801. \]
Since the alternate hypothesis is of the form \( \beta_1 \neq b, \) the \( P \)-value is the sum of the areas to the right of \( t = 29.801 \) and to the left of \( t = -29.801. \)
From the \( t \) table, \( P < 0.001. \) A computer package gives \( P = 2.37 \times 10^{-13} \).
The claim is not plausible.

(e) \( x = 0. \) The unloaded length is \( \hat{\beta}_0 + \hat{\beta}_1(0) = \hat{\beta}_0. \)

The null and alternate hypotheses are \( H_0: \beta_0 \geq 1.5 \) versus \( H_1: \beta_0 < 1.5. \)
\[ \hat{\beta}_0 = 6.6361, s_{\hat{\beta}_0} = 1.1455, t = (6.6361 - 1.5)/1.1455 = 4.484. \]
Since the alternate hypothesis is of the form \( \beta_0 < b, \) the \( P \)-value is the area to the left of \( t = 4.484. \)
From the \( t \) table, \( P > 0.40. \) A computer package gives \( P = 0.9997. \)
The claim is plausible.

7.3.4

4. (a) The slope is \(-0.13468. \)

(b) \( \hat{\beta}_1 = -0.13468, s_{\hat{\beta}_1} = 0.03798. \) Since \( n = 120 \) is large, use the \( z \) table to construct a confidence interval.
\[ z_{0.025} = 1.96. \) A 95% confidence interval is \(-0.13468 \pm 1.96(0.03798), \) or \((-0.2091, -0.0602). \)

(c) The null and alternate hypotheses are \( H_0: \beta_1 \geq -0.1 \) versus \( H_1: \beta_1 < -0.1. \)
\[ \hat{\beta}_1 = -0.13468, s_{\hat{\beta}_1} = 0.03798, z = (-0.13468 - (-0.1))/0.03798 = -0.91. \]
Since the alternate hypothesis is of the form \( \beta_1 < b, \) the \( P \)-value is the area to the left of \( z = -0.91. \)
From the \( z \) table, \( P = 0.1814. \)
It is plausible that the slope is greater than or equal to \(-0.1. \)
6. \( n = 40, \quad \sum_{i=1}^{n}(x_i - \bar{x})^2 = 98.775, \quad \sum_{i=1}^{n}(y_i - \bar{y})^2 = 19.10, \quad \bar{x} = 26.36, \quad \bar{y} = 0.5188,\)
\(\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = 826.94.\)

(a) \( r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \cdot \sum_{i=1}^{n}(y_i - \bar{y})^2}} = 0.602052.\)

(b) The error sum of squares is \(\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = (1 - r^2) \sum_{i=1}^{n}(y_i - \bar{y})^2 = 12.177.\)

The regression sum of squares is \(\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 - \sum_{i=1}^{n}(y_i - \bar{y})^2 = 19.10 - 12.177 = 6.923.\)

The total sum of squares is \(\sum_{i=1}^{n}(y_i - \bar{y})^2 = 19.10.\)

(c) \(\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = 0.008371956\) and \(\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 0.298115.\)

The equation of the least-squares line is \(y = 0.298115 + 0.008371956x.\)

(d) \(0.298115 + 0.008371956(40) = 0.633\text{ mm}\)

(e) Let \(x\) be the required temperature. Then \(0.5 = 0.298115 + 0.008371956x,\) so \(x = 24.1^\circ C.\)

(f) No. If the actual amount of warping happens to come out above the value predicted by the least-squares line, it could exceed 0.5 mm.