STATS 452/652, Spring 2013

Midterm 1 (February 26, 2013)

Name:______________________________

Level:________

- Read assignments carefully
- If you do not see a solution — go to the next problem, good ideas may come later
- Show work
- Good Luck!
<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Total</th>
</tr>
</thead>
</table>
Problem 1. Find the quantile function for the following distributions:

a) [2 points] Exponential: \( F(x) = 1 - e^{-\lambda x}, \ x \geq 0 \).

b) [2 points] Uniform on the interval \([5, 7]\).

c) [2 points] \( F(x) = \frac{x^2 - 9}{7}, \ x \in [3, 4] \).
Problem 2. Find the 60% percentile of the following distributions

a) [2 points] Exponential with variance 25.

b) [2 points] Uniform with mean 5 and variance 1/3.

c) [2 points] \( F(x) = 1 - e^{x^2}, x \geq 0. \)
Problem 3. The taxicab waiting time $X$, measured in minutes, during rush hours at a major airport is described by a random variable with the following probability density function

$$f(x) = \begin{cases} 2x, & \text{if } 0 \leq x \leq T \\ 0, & \text{otherwise} \end{cases}$$

a) [2 points] Find the maximal waiting time $T$.

b) [2 points] Find the cumulative distribution function of $X$.

c) [2 points] What is the probability that waiting time is less than 30 sec?

d) [2 points] Find the mean (expected) waiting time.
Problem 4. [6 points] A 400 page book contains 200 sheets of paper. The thickness of the paper has mean 0.1 mm and standard deviation 0.02 mm. What is the probability that a randomly chosen book is less than 19.43 mm thick (w/o covers)?
Problem 5. Let $X_i, i = 1, \ldots, n$ be iid rvs with mean $m$ and standard deviation $\sigma$ and

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i.$$ 

a) [3 points] Find the mean $E(\bar{X})$

b) [3 points] Find the variance $Var(\bar{X})$
Problem 6. [6 points] Consider a random variable $X$ with cumulative distribution function $F(x)$. Show that $Y = F(X)$ is a uniform random variable on $[0, 1]$. 
**Problem 7.**

Figure 1 shows empirical cdfs for data sets A (left) and B (right) with the standard Normal fit.

a) [4 points] Compare the empirical medians $\mu_A$ and $\mu_B$ for the two data sets. Explain your answer.

b) [2 points] How many observations are in the data set A?

---

Figure 1: Graphs for **Problem 7.**
Problem 8: 652 students only [6 points] Let $X$ be a rv with the cdf $F(x) = 1 - e^{-x^2}$, $x \geq 0$. Find the cdf, pdf, mean, and variance of the rv $X^2$. 