All notations are from lectures. Bold numbers are only for STAT653 students.

1.1 Show that \( \text{Var}(X) = E[X^2] - E[X]^2 \).

1.2 Show that \( \sum_k f(k) = 1 \), where \( f(k) \) is the pmf of the following distributions: a) Bernoulli, b) Binomial, c) Geometric, and d) Poisson.

1.3 Show that for a Binomial r.v. \( Y \) with parameters \((n, p)\)

\[
P(Y = k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}.
\]

1.4 Find the average number of roulette spins necessary to get the number 17. (Assume American roulette – with 36 numbers and two zeros).

1.5 Find the average loss in a roulette game if one bets on a number, on four numbers. The answer should be given in % to the bet amount. (Assume American roulette – with 36 numbers and two zeros).

1.6 The probability for a complication after a particular type of surgery is \( p = 0.005 \).

a) Find the probability that there will be at least one complication in 100 of such surgeries. b) Find the average number of complications after 100 surgeries. c) Find the probability that there will be at least one complications after 1000 surgeries. [Hint: Use a Poisson approximation.]

1.7 Use the definition of the mean to find the mean of a Poisson random variable with parameter \( \lambda \).

1.8 Prove the linear property of the expectation: if \( Y = aX + b \), where \( X \) is a r.v. and \( a, b \) are constants, then \( E(Y) = a \ E(X) + b \).