All notations are from lectures. Bold numbers are only for STAT653 students.

3.1 [Discussed in class.] Prove that if \( X_i \sim \text{Bernoulli}(p) \), \( i = 1, 2, \ldots \), and \( N \sim \text{Poisson}(\lambda) \), then
\[
\sum_{i=1}^{N} X_i \sim \text{Poisson}(\lambda p).
\]

3.2 Find the pgf, mgf, expected value, and \( p_0 \) for the rv \( X \) defined as the number of successes in \( N \) independent Bernoulli trials with the same success probability \( p \) if \( N \) has the following distribution a) Poisson; b) Geometric; c) Negative binomial; d) Bernoulli.

3.3 Compare and discuss the expected values and the probabilities \( p_0 \) (the probability to have no successes) found in the problem 3.2, items a) - c), assuming that the expected number of successes is fixed.

3.4 Prove the moment extraction formula, for the \( k \)-th moment \( m_k \) and the mgf \( M(s) \):
\[
m_k = \frac{d^k}{ds^k} M(s) \bigg|_{s=0}.
\]

3.5 Show that the variance of a rv \( X \) with pgf \( G(z) \) is given by
\[
\text{Var}(X) = G''(1) + G'(1) - [G'(1)]^2.
\]

3.6 Use the mgf approach to find the variance of the following distributions: a) Geometric; b) Poisson.

3.7 Let \( X_i \) be iid rvs with a common mgf \( M_X(z) \) and \( Y = \sum_{i=1}^{n} X_i \). Show that the mgf of \( Y \) is given by
\[
M_Y(z) = [M_X(z)]^n.
\]

3.8 Prove that a series of Binomial distributions with parameters \((n_i, p_i)\) such that \( n_i p_i = \lambda, n_i \to \infty \) converges to the Poisson distribution with parameter \( \lambda \). [Hint: Use convergence of the corresponding PGFs.]
3.9 We will call $N$-strategy the following rule of playing a roulette game: One covers $N$ distinct numbers every game, betting $1 on each number, or $N$ total. Find the payoff distribution, mean, and variance for an $N$-strategy. Discuss the similarities and differences of different strategies. Suggest the “best” strategy and support your choice.