Below we assume that $Z_t \sim WN(0, \sigma^2)$.

**Problem 1**
Consider MA(1) process $X_t = a Z_t + b Z_{t-1}$. Find the white noise $W_t$ such that the process $X_t$ is presented as $X_t = W_t + \theta W_{t-1}$ with $W_t \sim WN(0, \gamma^2)$.

**Problem 2**
Find acvf and acf for MA(1), MA(2), MA(3).

**Problem 3**
Show that the MA(1) processes $X_t = Z_t + \theta Z_{t-1}$ and $X_t = Z_t + 1/\theta Z_{t-1}$ have the same acf.

**Problem 4**
Find MA($q$) representation for the stationary process with

\[
\begin{align*}
\text{a) acf } &\rho(h) = \begin{cases} 
1, & h = 0, \\
0.3, & |h| = 1, \\
0, & |h| > 1.
\end{cases} \\
\text{b) acvf } &\gamma(h) = \begin{cases} 
31/18, & h = 0, \\
-35/36, & |h| = 1, \\
1/6, & |h| = 2, \\
0, & |h| > 2.
\end{cases}
\end{align*}
\]

**Problem 5**
Find the acf of the following MA(2) processes:

\[
\begin{align*}
\text{a) } &X_t = Z_t + 0.7 Z_{t-1} - 0.2 Z_{t-2}; \\
\text{b) } &X_t = Z_t + 0.3 Z_{t-1} - 0.4 Z_{t-2}.
\end{align*}
\]

**Problem 6**
Consider the infinite-order MA process $\{X_t\}$ defined by

\[X_t = Z_t + C(Z_{t-1} + Z_{t-2} + \ldots),\]

where $C$ is a constant. Show that the process $X_t$ is non-stationary. Show that the process $Y_t = \nabla X_t$ is stationary and find its acf.

**Problem 7**
Find a stationary process with the autocorrelation function $\rho(h) = (-1)^{|h|}$.

**Problem 8**
Check invertibility for the following processes:

\[
\begin{align*}
\text{a) } &X_t = Z_t - 0.2 Z_{t-1}; \\
\text{b) } &X_t = Z_t + 1.5 Z_{t-1}; \\
\text{c) } &X_t = Z_t - 2.5 Z_{t-1} + Z_{t-2}; \\
\text{d) } &X_t = Z_{t-2} - 2 Z_{t-5}.
\end{align*}
\]
Problem 9
Check that the following processes are invertible. Find the first three coefficients in their MA(∞) representation:
   a) \( X_t = Z_t + 0.5 Z_{t-1} \);
   b) \( X_t = Z_t - 0.2 Z_{t-1} \).

Problem 10
Find the range of \( \alpha \) for which the following process is invertible:
\[
X_t = 3\alpha Z_t - (3 + \alpha) Z_{t-1} + Z_{t-2}.
\]