We assume below that $Z_t$ is a white noise with mean 0 and variance $\sigma_Z^2$.

3.1 Find the acf of the second-order MA processes

a) $X_t = Z_t + 0.7Z_{t-1} - 0.2Z_{t-2}$

b) $X_t = Z_t + 0.3Z_{t-1} - 0.4Z_{t-2}$

3.2 Show that the acf of the MA process

$$X_t = \frac{1}{m+1} \sum_{k=0}^{m} Z_{t-k}$$

is given by

$$\rho(k) = \begin{cases} 
(m+1-k)/(m+1), & |k| \leq m, \\
0, & |k| > m.
\end{cases}$$

3.3 Show that the MA process

$$X_t = Z_t + C(Z_{t-1} + Z_{t-2} + \ldots)$$

where $C$ is a constant is non-stationary. Show also that $\nabla X_t$ is a MA(1) process and
it is stationary. Find the acf of $\nabla X_t$.

3.4 Find a sequence of random variables (stochastic process) with the following autocovariance function:

a) $\rho(k) = (-1)^{|k|}$,

b) $\rho(k) = \begin{cases} 
1, & k = 0, \\
0.4, & k = \pm 1, \\
0, & \text{otherwise}.
\end{cases}$

3.5 Show that there is no stationary stochastic process with acf

$$\rho(k) = \begin{cases} 
1, & k = 0, \\
0.7, & k = \pm 1, \\
0, & |k| > 1.
\end{cases}$$