We assume below that \( Z_t \) is a white noise with mean 0 and variance \( \sigma_Z^2 \).

9.1 Consider ARMA(2,2) model

\[
X_t = -X_{t-1} - 2/9 X_{t-2} + Z_t - 5/6 Z_{t-1} + 1/6 Z_{t-2}.
\]

a) Use prediction operator to construct 3-step forecast,
b) Use \( \pi \)-weights to construct 3-step forecast.
Discuss how the prediction operator forecast formula is related to \( \pi \)-weights forecast formula.
c) Use \( \psi \)-weights to find error of 3-step forecast, find the error variance.
d) Compare the variance of the 3-step forecast error with variance of the \( X_t \) (estimate the latter using \( \psi \)-weights). Discuss.
e) Find minimal \( h \) such that the variance of \( h \)-step forecast error is more than 90% of the variance of \( X_t \) (forecast for more than \( h \) steps is practically useless).

9.2 [Chatfield, Ex. 5.6] Consider the ARIMA(0,1,1) process

\[
(1 - B)X_t = (1 - \theta B)Z_t.
\]

Show that \( \hat{x}_t(1) = x_t - \theta Z_t \), and \( \hat{x}_t(h) = \hat{x}_t(h - 1) \) for \( h \geq 2 \). Express \( \hat{x}_t(1) \) in terms of \( x_t \) and \( \hat{x}_{t-1}(1) \) and show that this is equivalent to exponential smoothing. Consider \( \psi \)-weights and show that the variance of the \( h \)-step forecast error is

\[
\left[ 1 + (h - 1)(1 - \theta)^2 \right] \sigma_Z^2.
\]