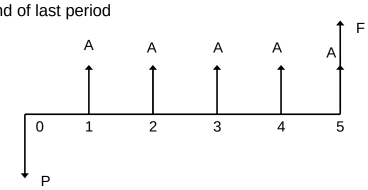
Chapter 4
More Interest Formulas

- Uniform Series Compound Interest
  - Uniform payment (or income): e.g., car loans, house mortgage, monthly pay check
  - Convention: P – at beginning; A – end of each period; F – end of last period



Uniform Series: A to F

$$F = A\left[\frac{(1+i)^n - 1}{i}\right] = A(F/A, i\%, n)$$

- Example 4-1

  Deposit \$500 at the end of each year for 5 years. Interest rate is 5% compounded annually. How much in the bank immediately after the last (the 5^{th}) deposit?

Uniform Series: A to P

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = A(P/A, i\%, n)$$

- Example 4-4

  A = \$140; n = 5 year * 12 = 60 months; i = 1% per month

Option 1: Pay \$6800 today;
Option 2: Pay \$140 each month for 5 years.

Example

Year	Cash Flow
1	+100
2	+100
3	+100
4	0
5	-F

Calculate F=? assume i = 15%

Uniform Series: P to A

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = P(A/P, i\%, n)$$

- Example 4-3

  Borrow $5000 and will make 5 equal payments at the end of each year. Interest rate is 6%. How much each payment should be?

Uniform Series: F to A

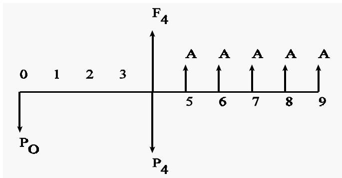
$$A = F\left[\frac{i}{(1+i)^n - 1}\right] = F(A/F, i\%, n)$$

- Example 4-2

  Need $1000 at the end of the year. Interest rate is 6%, compounded monthly. How much to deposit each month?

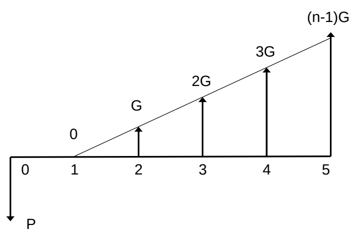
Deferred Annuities

- Number of annuity payments is less than the number of analysis periods



- Example 4-6

Arithmetic Gradient Series



Arithmetic Gradient Series: G to P

$$P = G\left[\frac{(1+i)^n - in - 1}{i^2(1+i)^n}\right] = G(P/G,i,n)$$

- Example 4-8

  Maintenance cost for a car in the first five years is shown below:

1	\$120
2	\$150
3	\$180
4	\$210
5	\$240

  Assume interest is 5%, how much money should be put in bank now to cover the costs for the first five years

Example with Negative G

- Example

  Maintenance cost each year is shown below:

1	$24,000
2	$18,000
3	$12,000
4	$6,000

  Assume interest is 10%, how much money should be put in bank now to cover the costs for the four years

Arithmetic Gradient Series: G to A

$$A = G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right] = G(A/G, i, n)$$

- Example 4-9

  Maintenance cost each year is shown below:

1	$100
2	$200
3	$300
4	$400

  Assume interest is 6%, what is the equivalent uniform annual maintenance cost

Example with Negative G

- Example

  Maintenance cost each year is shown below:

1	$24,000
2	$18,000
3	$12,000
4	$6,000

  Assume interest is 10%, what is the equivalent annual maintenance cost?

factor name	converts	symbol	formula
single payment compound amount	P to F	$(F/P, i\%, n)$	$(1+i)^n$
single payment present worth	F to P	$(P/F, i\%, n)$	$(1+i)^{-n}$
uniform series sinking fund	F to A	$(A/F, i\%, n)$	$\frac{i}{(1+i)^n - 1}$
capital recovery	P to A	$(A/P, i\%, n)$	$\frac{i(1+i)^n}{(1+i)^n - 1}$
uniform series compound amount	A to F	$(F/A, i\%, n)$	$\frac{(1+i)^n - 1}{i}$
uniform series present worth	A to P	$(P/A, i\%, n)$	$\frac{(1+i)^n - 1}{i(1+i)^n}$
uniform gradient present worth	G to P	$(P/G, i\%, n)$	$\frac{(1+i)^n - 1}{i^2(1+i)^n} - \frac{n}{i(1+i)^n}$
uniform gradient future worth	G to F	$(F/G, i\%, n)$	$\frac{(1+i)^n - 1}{i^2} - \frac{n}{i}$
uniform gradient uniform series	G to A	$(A/G, i\%, n)$	$\frac{1}{i} - \frac{n}{(1+i)^n - 1}$

Non Conventional Problem

- Example
  At 6% interest rate, how much should be deposited at the start of each year for ten years (10 deposits) in order to empty the fund by drawing out $200 at the end of each year for ten years (10 withdrawals)

Nominal and Effective Interest

- Example 4-13: $100 deposit with interest rate of 5%, compounded semiannually

- Nominal Rate (r): without compounding
- Effective Rate (i_a): with compounding effect
- Both are usually expressed on *annual* bases

Nominal and Effective Interest

$$i_a = (1+i)^m - 1$$

$$i = \frac{r}{m}$$

i = effective interest rate per interest period
m = number of compounding periods per year

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Nominal and Effective Interest

Example 4-15:
"If I give you $50 on Monday, you owe me $60 the following Monday"
(a) r = ?
(b) i_a = ?
(c) F = ? at the end of 2 years?

Example 4-16:
Deposit *P* = $5,000; *r* = 8%; compounded quarterly (*m* = 4);
Calculate: 5 equal withdrawals at the end of each year (A = ?)

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Spreadsheet Applications

Example:

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