Chapter 4
More Interest Formulas

- Uniform Series Compound Interest
  - Uniform payment (or income): e.g., car loans, house mortgage, monthly pay check
  - Convention: P – at beginning; A – end of each period; F – end of last period

\[
F = A \left[\frac{(1+i)^n - 1}{i}\right] = A(F / A, i\%, n)
\]

- Example 4-1
  Deposit $500 at the end of each year for 5 years. Interest rate is 5% compounded annually. How much in the bank immediately after the last (the 5th) deposit?

\[
P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = A(P / A, i\%, n)
\]

- Example 4-4
  \[A = 140; \ n = 5 \text{ year} \times 12 = 60 \text{ months}; \ i = 1\% \text{ per month}\]
  - Option 1: Pay $6800 today;
  - Option 2: Pay $140 each month for 5 years.
Example

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+100</td>
</tr>
<tr>
<td>2</td>
<td>+100</td>
</tr>
<tr>
<td>3</td>
<td>+100</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>-F</td>
</tr>
</tbody>
</table>

Calculate F=? assume i = 15%

Uniform Series: P to A

\[
A = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] = P \left( A / P, i\%, n \right)
\]

Example 4-3

Borrow $5000 and will make 5 equal payments at the end of each year. Interest rate is 6%. How much each payment should be?

Uniform Series: F to A

\[
A = F \left[ \frac{i}{(1+i)^n - 1} \right] = F \left( A / F, i\%, n \right)
\]

Example 4-2

Need $1000 at the end of the year. Interest rate is 6%, compounded monthly. How much to deposit each month?
Deferred Annuities

- Number of annuity payments is less than the number of analysis periods

Example 4-6

Arithmetic Gradient Series

\[ P = G \frac{(1+i)^n - in - 1}{i^2(1+i)^n} \]

Example 4-8

Maintenance cost for a car in the first five years is shown below:

<table>
<thead>
<tr>
<th>Year</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$120</td>
</tr>
<tr>
<td>2</td>
<td>$150</td>
</tr>
<tr>
<td>3</td>
<td>$180</td>
</tr>
<tr>
<td>4</td>
<td>$210</td>
</tr>
<tr>
<td>5</td>
<td>$240</td>
</tr>
</tbody>
</table>

Assume interest is 5%, how much money should be put in bank now to cover the costs for the first five years.
Example with Negative G

Example

Maintenance cost each year is shown below:

1. $24,000
2. $18,000
3. $12,000
4. $6,000

Assume interest is 10%, how much money should be put in bank now to cover the costs for the four years?

Arithmetic Gradient Series: G to A

\[ A = G \left[ \frac{(1+i)^n - in - 1}{i(1+i)^{n-1} - i} \right] = G(A/G,i,n) \]

Example 4-9

Maintenance cost each year is shown below:

1. $100
2. $200
3. $300
4. $400

Assume interest is 6%, what is the equivalent uniform annual maintenance cost?
Non Conventional Problem

- **Example**
  At 6% interest rate, how much should be deposited at the start of each year for ten years (10 deposits) in order to empty the fund by drawing out $200 at the end of each year for ten years (10 withdrawals).

Nominal and Effective Interest

- **Example 4-13**: $100 deposit with interest rate of 5%, compounded semiannually
  - **Nominal Rate** ($r$): without compounding
  - **Effective Rate** ($i_a$): with compounding effect
  - Both are usually expressed on *annual* bases.
Nominal and Effective Interest

\[ i_a = (1 + i)^m - 1 \]
\[ i = \frac{r}{m} \]

\( i \) = effective interest rate per interest period
\( m \) = number of compounding periods per year

Example 4-15:
"If I give you $50 on Monday, you owe me $60 the following Monday"
(a) \( r = ? \)
(b) \( i_a = ? \)
(c) \( F = ? \) at the end of 2 years?

Example 4-16:
Deposit \( P = $5,000; \ r = 8\% \); compounded quarterly \( (m = 4) \);
Calculate: 5 equal withdrawals at the end of each year \( (A = ?) \)

Spreadsheet Applications

Example: