Queue Length Models for All-Way Stop-Controlled Intersections

Zong Tian and Michael Kyte

The analytical procedures for all-way stop-controlled intersections in the 2000 edition of the *Highway Capacity Manual* (HCM) lack a model to estimate the 95th percentile queue length. This is considered a major shortcoming of the procedures, since queue length is one of the key parameters used in the traffic engineering design process. This paper provides an assessment of three queue length models for potential application at all-way stop-controlled intersections. One is the theoretically based queue length model included in the HCM for two-way stop-controlled intersections, and the others are empirically based models: a model based on simulation and a newly calibrated model based on field data. The three models were compared on the basis of field data from 18 sites. All the models produce queue length results that closely match the field data. The queue length model for two-way stop-controlled intersections was developed with a sound theoretical basis; therefore, it is recommended that this model be adopted for analyzing all-way stop-controlled intersections in the HCM. However, the empirical models are easy to use and proved to be of the same level of accuracy. The empirical models can be used as an alternative, especially when a quick estimation of the queue length is desired.

Chapter 17 of the 2000 edition of the *Highway Capacity Manual* (HCM) (*1*) contains models and procedures for determining capacity and level of service for unsignalized intersections. While the procedures for all-way stop-controlled (AWSC) intersections include capacity and delay estimations, no queue length estimation method is provided. The lack of queue length estimation in the procedure is considered a major shortcoming, since queue length has been regarded as one of the key parameters in the traffic engineering design process. The primary objective of this paper is to assess available models and recommend a queue length model that can be used in analyzing AWSC intersections.

The focus of this paper is on evaluating two available queuing models. One is an empirical model developed by Tian et al. (*2*), and the other is the queue length model for analyzing two-way stop-controlled (TWSC) intersections included in the HCM.

QUEUE LENGTH MODELS

An empirical model based on simulation that provides an estimation of the 95th percentile queue length at AWSC intersections has been developed in several earlier studies (*2–5*). First, the well-known Little formula (*6*, p. 11) from queuing theory was validated on the basis of field data at unsignalized intersections, including AWSC and TWSC intersections. Little's formula establishes the relationship between average queue length, demand, and average delay for any type of queuing system. Since unsignalized intersections can generally be represented by an M/G2/1 model (*7*), the average queue at a stop-controlled intersection approach or an individual lane is directly related to the traffic demand and the average delay as shown in Equation 1:

$$L = \frac{V}{3{,}600} d \qquad (1)$$

where

L = average queue length (vehicles),
V = traffic demand [vehicles per hour (vph)], and
d = average delay (seconds per vehicle).

Equation 1 is essentially Little's formula from queuing theory, and its validity for application at unsignalized intersections has been documented in previous studies (*3, 5*).

The empirical model developed by Tian et al. (*2*) for AWSC intersections is shown in Equation 2. It calculates the 95th percentile queue length, $L_{95\%}$, directly from the average queue length, L:

$$L_{95\%} = 1.3L + 2.1\sqrt{L} + \frac{L}{L + 4.6} \qquad (2)$$

The empirical queue length model was validated with data collected as part of NCHRP Project 3-46, Capacity and Level of Service at Unsignalized Intersections (*8, 9*). The data set included a total of 18 AWSC intersections with more than 500 samples of 15-min-interval data.

The queue length model included in the HCM for TWSC intersections is shown in Equation 3:

$$L_{95\%} = 900T\left[\frac{V}{c} - 1 + \sqrt{\left(\frac{V}{c} - 1\right)^2 + \frac{\left(\frac{3{,}600}{c}\right)\left(\frac{V}{c}\right)}{150T}}\right]\frac{c}{3{,}600} \qquad (3)$$

where c is the capacity for the subject approach or a lane (vph) and T is the analysis period (h).

When an analysis based on the HCM's default's 15-min period is performed, $T = 0.25$ h, and Equation 3 becomes

$$L_{95\%} = 225\left[\frac{V}{c} - 1 + \sqrt{\left(\frac{V}{c} - 1\right)^2 + 96\frac{V}{c^2}}\right]\frac{c}{3{,}600} \qquad (4)$$

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The queue length model shown in Equation 4 is based on a study by Wu (*7*). The model is a combination of the M/G2/1 queuing model for undersaturated conditions and the empirical queue transformation model for oversaturated conditions. This queue length model (Equation 4) closely resembles the delay model as shown in Equation 5 for TWSC intersections in the HCM.

$$d = \frac{3,600}{c} + 900T\left[\frac{V}{c} - 1 + \sqrt{\left(\frac{V}{c} - 1\right)^2 + \frac{\left(\frac{3,600}{c}\right)\left(\frac{V}{c}\right)}{450T}}\right] + 5 \qquad (5)$$

One reasonable assumption is that AWSC intersections possess characteristics similar to TWSC intersections from the point of view of queuing systems, where vehicle arrivals follow a random process, and the service time can be represented by a general (G) distribution. Therefore, the same queue length model for TWSC intersections might be used in analyzing AWSC intersections.

The following section presents the results of application of both models at AWSC intersections and a comparison of these results with the field data.

MODEL COMPARISON AGAINST FIELD DATA

The data set from NCHRP Project 3-46 was used to compare the two queue length models described in the previous section. In the following discussion, the empirical model developed by Tian et al. will be denoted as T1, and the queue length model for TWSC intersections in the HCM will be denoted as HCM. Essentially, the same results of the T1 model from earlier studies (*2–4*) are reproduced here for the purpose of easy illustration and comparison. Again, the data included a total of 18 intersections, among which 11 had single-lane approaches and seven had multilane approaches. Data were aggregated on the basis of 15-min periods, consistent with the analysis period used in the HCM. To obtain the 95th percentile queue length, the queue length on an approach or a lane must be continuously sampled on the basis of a specified time interval. In this case, the queue length was sampled once every 20 s, and the 95th percentile queue during a 15-min period was determined on the basis of the 95th percentile value of all the samples. For example, a 15-min period consists of 45 intervals of 20 s. The queue lengths at the end of each 20-s intervals were collected. The queue length was then sorted in ascending order, and the 95th percentile queue is the 86th value ($90 \times 95\% = 86$) in the order. Figure 1 shows the distribution of the 95th percentile queue at all the 18 sites. As can be seen from Figure 1, the majority of the queue length measurements are less than five vehicles, which indicates that it is difficult to find AWSC intersections with long queues in the field. When the 95th percentile queue length exceeds five vehicles, the site may have already met the signal warrants and been signalized.

Calculating the 95th percentile queue length with the HCM queue length model requires the traffic demand and the capacity for an approach or an individual lane. The capacity was estimated directly from the field data on the basis of the following equation:

$$c = \frac{3,600}{h_s} = \frac{3,600}{t_s + t_{mv}} \qquad (6)$$

where

h_s = saturation headway (s),
t_s = service time (s), and
t_{mv} = move-up time (s).

Equation 6 provides an estimate of the capacity of an approach or a lane by using the field-measured service time, t_s, and the move-up time, t_{mv}. As shown in Figure 2, the service time represents the average time that a vehicle spends at the stop line position during the analysis period, and the move-up time represents the average time it takes for the following vehicle to move to the stop line after the previous vehicle departs from the stop line. The sum of the service time and the move-up time is the saturation headway as defined in the HCM for AWSC intersections. The capacity estimation method presented in Equation 6 has been used extensively in previous studies (*8–10*), and it proved to be a reliable method for estimating capacity under conditions of noncontinuous queuing.

Figure 3 shows the results based on the T1 model, and Figure 4 shows the results based on the HCM model. In both figures, the results from the models were compared with the field data, where the mean

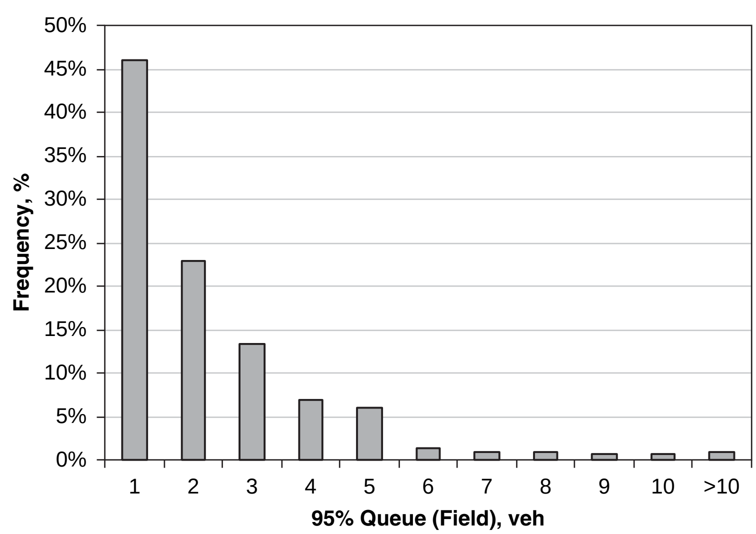

FIGURE 1 Distribution of queue length data.

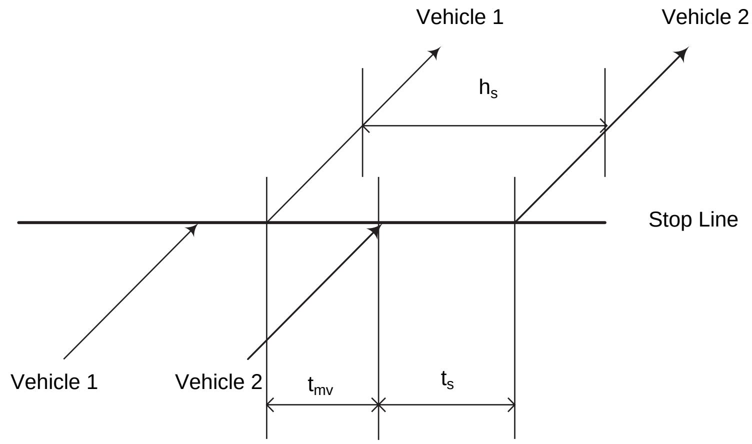

FIGURE 2 Service time and move-up time at unsignalized intersections.

absolute error (MAE) and mean absolute percent error (MAPE) were calculated for both models on the basis of Equations 7 and 8:

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left|L_{mi} - L_{fi}\right| \quad (7)$$

$$\text{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|L_{mi} - L_{fi}\right|}{L_{fi}} \quad (8)$$

where

L_{mi} = queue length of data sample i from the model,
L_{fi} = queue length of data sample i from the field, and
n = total number of data samples.

Both MAE and MAPE are good indicators of the model accuracy and have been used in previous studies (*8, 9*). However, both MAE and MAPE are considered empirical measures, since whether they are acceptable in practice will depend on the specific situation and the user's judgment. In this case, when MAE is less than 1.0, the average error of estimation is less than one vehicle, which is considered good from the practical point of view. MAPE could yield high values (e.g., 40%) when the queue length is small but may be still considered acceptable since the MAE is small.

As shown in Figure 3 and Figure 4, both models yielded results that closely matched the field data. The T1 model produced slightly better results than the HCM model, as indicated by the smaller MAE and MAPE values (0.44 versus 0.78 and 23% versus 40%, respectively). The higher MAPE values reflect the fact that most of the queue length data are in the range of one to five vehicles (see Figure 1), where a small numeric error will result in high percentage errors. However, from a practical point of view, both models provide acceptable queue length estimation. While the HCM model possesses sound theoretical background, the T1 model is much easier to use, requiring either the average queue length or the demand and average delay.

A further examination of the T1 model indicates that the third term of the equation has only a marginal effect on the final queue length calculation. The model was recalibrated after the third term was removed from the equation, and the following new equation was obtained through regression analysis:

$$L_{95\%} = 1.3L + 2.3\sqrt{L} \quad (9)$$

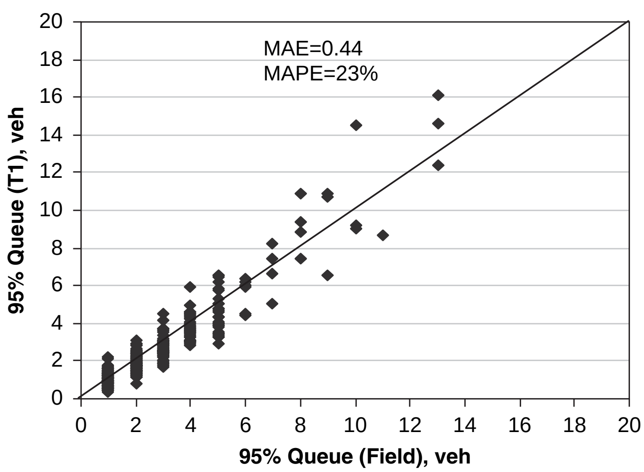

FIGURE 3 Comparison of T1 model with field data.

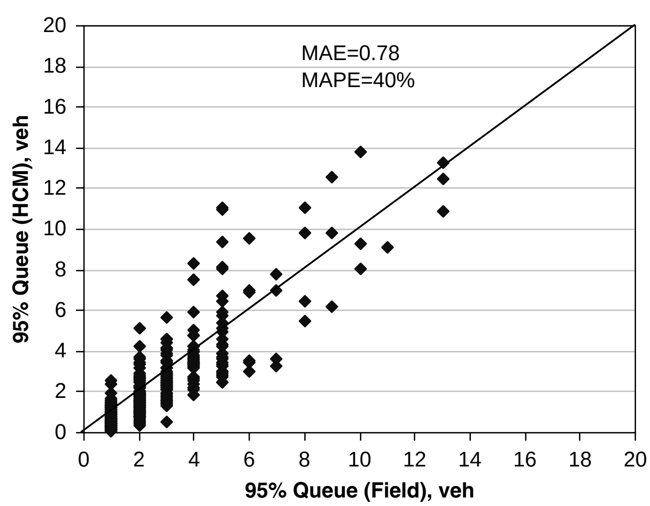

FIGURE 4 Comparison of HCM model with field data.

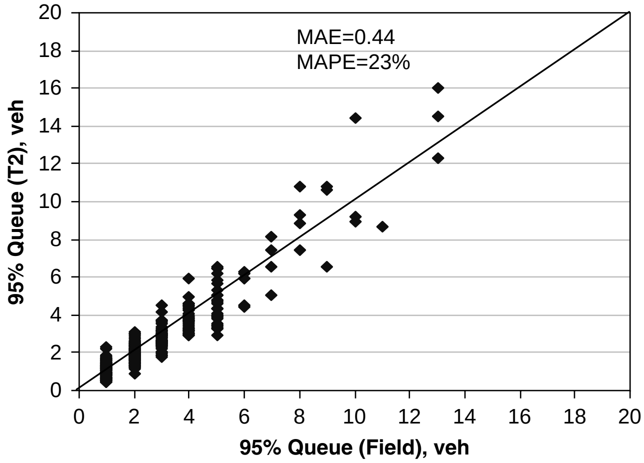

FIGURE 5 Comparison of T2 model with field data.

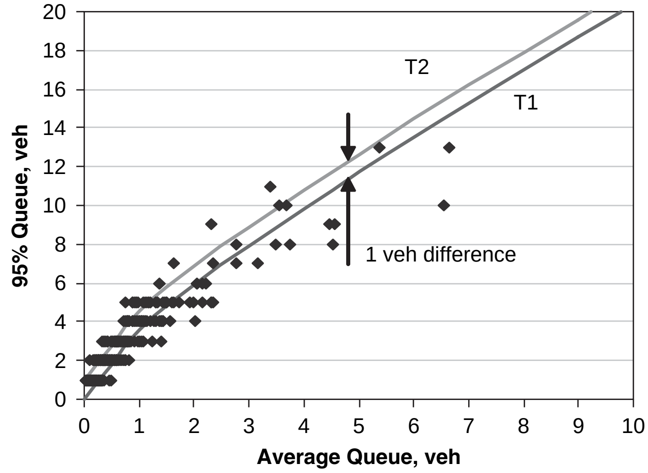

FIGURE 6 Model fit with field data.

Figure 5 illustrates the results with the newly calibrated model (denoted as T2). As the figure shows, the T2 model produced results similar to the T1 model, with the same MAE of 0.44 and the same MAPE of 23%.

Figure 6 shows the lines fit to the field data for the two models T1 and T2. For better illustration, one vehicle length was added to the T2 curve to create an offset view of the two curves. As can be seen, the two models basically produced identical results.

SUMMARY AND CONCLUSIONS

This paper evaluated three queue length estimation models for application at AWSC intersections: the theoretical HCM model for TWSC intersections, an earlier empirical model based on simulation (T1), and a newly calibrated simplified model (T2). The results of all three models were compared with the field data collected as part of NCHRP Project 3-46. The results of the study lead to the following conclusions:

- All three models produce accurate queue length results. On the basis of comparisons with field data, all the models produced queue length results with an error of less than one vehicle.
- The HCM model was developed on the basis of sound queuing theory; therefore, it should be adopted as a formal model for queue length estimation at AWSC intersections.
- The empirical model from this study is easy to use, and it is still highly transferable because the average queue length is the only variable required in the equation. Similar results are expected at other AWSC intersections. Therefore, the empirical model can be used as an alternative, especially when a quick estimation of the queue length is desired or field measurement and model calibration are being performed.
- In using the empirical model developed in this study, caution must be exercised with regard to its valid range. The empirical model in this study was derived on the basis of field observations of 95% queue length of less than 14 vehicles. The model could overestimate the queue length when this range is exceeded; however, no field data could support this hypothesis.

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