Modification of Webster’s Minimum Delay Cycle Length Equation Based on HCM 2000

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ABSTRACT

The famous British transportation researcher, F. V. Webster, developed a series of useful traffic theories, which have had a very big influence on the modern traffic analysis since the 1950s. However, based on this study, Webster’s minimum delay cycle length equation overestimates the optimal cycle length compared to the results based on the HCM 2000 method. This is due to the restructuring of the HCM 2000 delay equation as compared to the original Webster’s delay calculation. For an isolated intersection, based on Webster’s delay equation, the delay will become infinity when the degree of saturation of a lane group approaches one, which is unrealistic, while the delay based on HCM 2000 method can accommodate some random failures and short-term oversaturation situations. The HCS software was used to conduct experiments for a typical four-phase intersection over a wide range of volume and lost time scenarios, and the results were used to modify the original Webster minimum delay cycle length equation. The new minimum delay cycle length equations based on this study significantly improve the accuracy of predicting the optimal cycle length for isolated intersections at high traffic volume conditions.

Key words: Optimal Cycle Length, Isolated Intersection, Delay, HCM
INTRODUCTION

In the 1950s, Webster conducted a series of experiments on pretimed isolated intersection operations (1). Two traffic signal timing strategies came from his study. One is signal phase splits. Webster demonstrated, both theoretically and experimentally, that pretimed signals should have their critical phases timed for the equal degrees of saturation for a given cycle length to minimize the delay. The other is the minimum delay cycle length equation, which is shown as Equation 1. In developing the equation for the optimal minimum delay cycle length, it was assumed that the effective green times of the phases were in the ratio of their respective y values (flow ratios).

\[
c_0 = \frac{1.5L + 5}{1 - Y}
\]

where \( c_0 \) = the optimal minimum delay cycle length, sec;
\( L \) = total lost time within the cycle, sec; and
\( Y \) = the sum of critical phase flow ratios (2).

The above two strategies are very useful for traffic design and planning. When the two rules are applied together, one can practically minimizes the resulting delay at an isolated pretimed signalized intersection. However, when the traffic demand of an intersection is high, which causes a high value of degrees of saturation, the optimal cycle length based on Webster’s equation will become extremely high, may be 30 to 40 seconds higher than the value based on the HCM 2000 delay calculation. The optimal cycle length overestimation of Webster’s equation has not been addressed yet based on our literature reviews. The purpose of this paper is to find out the reason for the higher cycle length prediction by Webster equation and provide more accurate models.

The paper is organized as follows. A series of experiments were conducted on a hypothetical isolated pretimed four-leg traffic signal by using the HCS software and an Excel spreadsheet model implementing Webster’s equation. The optimal cycle lengths for different traffic demand situations were calculated based on both HCM method and Webster’s equation. Comparisons were made and alternatives proposed. Finally, a summary and conclusions were provided.

THEORETICAL BACKGROUND

The optimal cycle length, which gives the minimum average control delay experienced by all vehicles that arrive in the analysis period, are closely related to the delay calculation methodologies. If the delay results calculated from the Webster and HCM 2000 methods are different, the optimal cycle lengths will most likely be different as well.

Webster Delay Equation

The delay calculation for the Webster method is expressed as Equation 2:

\[
d = \frac{c(1 - \lambda )^2}{2(1 - \lambda x)} + \frac{x^2}{2q(1 - x)} - 0.65\left(\frac{c}{q^2}\right)^3 x^{(2 + 5\lambda )}
\]

where \( d \) = average delay per vehicle on the particular lane group of the intersection, sec/veh;
\( c \) = cycle length, sec;
\( q \) = flow, vehicles/sec;
\( \lambda = \text{proportion of the effective green with respect to cycle length (i.e. } g/c \text{ and } g \text{ is effective green, sec); and} \)

\( x = \text{the degree of saturation. This is the ratio of the actual flow to the maximum flow which can be passed through the intersection from this lane group, and is given by } x = g/\lambda s, \text{ where } s \text{ is the saturation flow in vehicles per second.} \)

The first term of Equation 2 represents the delay when the traffic is assumed to be arriving uniformly. The second term of the equation makes some allowance for the random nature of the arrivals. It is an expression for the delay experienced by vehicles arriving randomly in time at a “bottleneck”, queueing up, and leaving at constant headways. The third term of the equation is an empirical correction term to give a closer fit for all values of flow. Normally, the last term is relatively small comparing to the total delay and frequently is omitted by reducing ten percent of the first two terms (3).

**HCM 2000 Delay Equation**

The average control delay per vehicle for a given lane group in the HCM 2000 is calculated by using the following equation

\[
d = d_1 \times PF + d_2 + d_3
\]

where  

\( d = \text{control delay per vehicle, s/veh}; \)

\( d_1 = \text{uniform control delay assuming uniform arrivals, s/veh}; \)

\( PF = \text{uniform delay progression adjustment factor, which accounts for effects of signal progression (in this paper, } PF = 1 \text{ because an isolated intersection is assumed);} \)

\( d_2 = \text{incremental delay to account for effect of random arrivals and oversaturation queues, adjusted for duration of analysis period and type of signal control; this delay component assumes no initial queue for a lane group at the start of analysis period, s/veh; and} \)

\( d_3 = \text{initial queue delay, which accounts for delay to all vehicles in analysis period due to an initial queue at the start of analysis period, s/veh. A zero initial queue is assumed in this paper.} \)

The equation used to calculate the uniform control delay, described in Equation 4, is essentially the same as the first term of Webster’s delay formulation and is widely accepted as an accurate depiction of delay for the idealized case of uniform arrivals. Note that degrees of saturation beyond 1.0 is not used in the computation of \( d_1 \).

\[
d_1 = \frac{0.50c \left(1 - \frac{g}{c}\right)^2}{1 - \min(1, x) \frac{g}{c}}
\]

where the terms in the equation are the same as defined before.

Equation 5 is used to estimate the incremental delay due to nonuniform arrivals and temporary cycle failures (random delay) as well as delay caused by sustained periods of oversaturation (oversaturation delay). The equation assumes that there is no unmet demand that causes initial queues at the start of the analysis period. The incremental delay term, \( d_2 \), is valid for all values of \( x \), including highly oversaturated lane groups.
\[ d_2 = 900T \left[ (x-1) + \sqrt{(x-1)^2 + \frac{8kIx}{CT}} \right] \]  

where  
- \( T \) = duration of analysis period, hour; 
- \( k \) = incremental delay factor that is dependent on actuated controller settings; 
- \( I \) = upstream filtering/metering adjustment factor; 
- \( C \) = lane group capacity, vph; and 
- \( x \) = lane group v/c ratio or degree of saturation.

There are significant differences between the second term of Webster’s delay equation and HCM 2000’s second term of delay calculation. When the degree of saturation is close to one, the delay based on the Webster’s equation will approach infinity, which is unrealistic. However, the HCM 2000 delay will be somewhat along the solid line of Figure 1 for saturated and oversaturated conditions.

The Level of Service is closely related to the average control delay of the intersection. For easy reference, the HCM 2000 Level of Service criteria based on the average control delay are listed in Table 1(4).

Optimal Minimum Delay Cycle Length Equation

For an isolated intersection, optimum cycle length is corresponding to the minimum total delay of the intersection. This minimum total delay situation can be obtained by selecting an appropriate cycle length and green splits. For a given cycle length, the effective green phases can be selected in proportion to the critical flow ratio of the phases. One way to obtain the optimal cycle length is to take the derivative of the expression for total delay of the intersection with respect to cycle length and set equal to zero. Because the delay calculations are different between the Webster and HCM 2000 method, as shown previously, one would expect that the optimal cycle length equation will be different. Figure 2 shows the relationship between cycle length and delay based on a sample intersection described in Webster’s paper (1). From the graph, one can see that the optimal cycle lengths corresponding to the minimum delay of the intersection are similar at the low traffic volume, i.e., 1600 vph. However, when the traffic volume is high, the optimal cycle lengths are significantly different. For example, the optimal cycle length from Webster is 40 seconds higher than that from the HCM 2000 when the volume is equal to 3000 vph.

Webster found the minimum cycle, \( c_m \), is just long enough to allow all the traffic which arrives in one cycle (assume uniform flow) to pass through the intersection in the same cycle, which can be expressed using Equation 6.

\[ c_m = L + \sum_{i=1}^{n} q_i c_m = \frac{L}{1 - Y} \]  

where \( q_i \) = the arrival volume at lane group i; and \( s_i \) = the saturation flow at lane group i.

Theoretically, the minimum cycle length will cause infinite delays because of the random nature of the traffic flow. Webster further developed the following linear approximation for the optimal cycle length for the practical application purposes:
\[ c_0 = \frac{KL + 5}{1 - Y} = c_m + \Delta c = \frac{L}{1 - Y} + \frac{0.5L + 5}{1 - Y} \]  
(7)

where \( K \) is a regression parameter. \( K \) is equal to 1.5 according to Webster which gives Equation 1. From the above theoretical analysis and experimental results shown in Figure 2, one would expect that Equation 1 should be modified correspondingly to accommodate the development of HCM 2000 delay equation.

**EXPERIMENTAL PROCEDURE**

In order to modify the Webster’s optimal cycle length equation, a series of experiments with a wide range of volume and lost time conditions were conducted. Since HCS 2000 does not have an optimization engine for the optimal cycle length, Synchro 5 (5) was used to derive the initial value of optimal cycle length and green splits. Because Synchro 5 and HCM 2000 are not completely compatible (6) in their capacity calculation procedures, the final optimal cycle length and green splits were either verified or changed by using gradient search methodology by HCS 2000. The same traffic volume, lost time and roadway conditions were input into an Excel spreadsheet and the optimal cycle lengths for each case were calculated using Webster’s optimal minimum delay cycle length equation. In order to compare the Webster and HCM 2000 results, the optimal cycle lengths from the two different methods were plotted together. The following shows the basic procedure used to conduct the experiments:

1. Start with a lower total lost time, i.e., 12 sec, and lower total volume, i.e., 1000 vph for a four-phase isolated intersection. Input the traffic data into Synchro 5. Optimize the green splits and cycle length equation using Synchro’s optimization tool.
2. Output Synchro’s data into HCS 2000 software.
3. Calculate the total control delay of the intersection by HCS 2000. For the same cycle length, conduct gradient search, i.e., increasing and decreasing five percent green splits, to find the minimum delay for different green splits.
4. Do the gradient search on the optimal cycle length, i.e., increasing or decreasing cycle length from Synchro by one or two seconds; and re-optimize the green splits for each new cycle length. Find the optimal cycle length corresponding to the minimum total delay from the HCS 2000.
5. Increase the traffic volume for the intersection. Repeat step 1 to 4 to find the optimal cycle length for the new traffic volume.
6. Repeat step 5 until the v/c ratio or degree of saturation approaches one.
7. Increase the total lost time to 14 seconds. Repeat step 1 to 6 to find the new optimal cycle lengths for each new lost time and traffic volume.
8. Repeat step 7 for the total lost time to 16, 18, and 20 seconds to get all the optimal cycle length for different traffic volumes and lost times using the HCM method.
9. Calculate optimal cycle length by using Webster’s optimal cycle length equation corresponding to same traffic volumes and lost times in steps 1 to 8.

**RESULTS AND NEW MINIMUM DELAY CYCLE LENGTH EQUATIONS**

The experiments were conducted over a wide range of volume and covers most lost time situations. Various parameters and results under different total lost time scenarios are shown in Tables 2 to 6, including arrival volumes, average control delays, degrees of saturation, \( X_{in} \), the sums of flow ratios, \( Y \), the HCM optimal cycle lengths, and the Webster’s optimal cycle lengths, \( c_0 \).
From the results shown in the tables, for the same lost time, the optimal cycle lengths from both HCM 2000 and Webster’s methods increase with the increasing of volumes and degrees of saturation. The optimal cycle lengths are similar for both HCM 2000 and Webster’s optimal cycle length equation under low volumes and degrees of saturation. However, at higher volumes and degrees of saturation situations, Webster optimal minimum delay cycle lengths are always higher than those from the HCM 2000 method. From the results of these tables, up to Level of Service C (delays less than or equal to 35 seconds), the residuals between the HCM 2000 and the Webster’s optimal cycle length equation are small. Thus, the Webster’s optimal cycle length equation is satisfactory for the Level of Service C or better. For the Level of Service D and worse, the Webster optimal cycle length equation clearly overestimates the results. The higher the volume and total lost time, the higher the overestimation.

In order to improve the Webster’s optimal cycle length equation, three regression models were proposed in this paper. The first one recalibrated the Webster’s minimum delay cycle length equation. The form of recalibrated Webster model is shown in Equation 8 (7).

\[
c_0 = \frac{aL + b}{1 - Y} \quad (8)
\]

where \(a\) and \(b\) are optimal minimum delay cycle length calculation coefficients. By using SPSS software (8), the \(a\) and \(b\) were obtained as 1.0 and 7.6, respectively. Therefore, the recalibrated Webster’s model is shown as Equation 9.

\[
c_0 = \frac{1.0L + 7.6}{1 - Y} \quad (9)
\]

In order to develop the second model, the optimal cycle lengths from HCM 2000 and Webster’s equation were plotted with the inverse of \(1/(1-Y)\). Figure 3, 4, 5, 6 and 7 show the plots for the total lost time of 12, 14, 16, 18 and 20 seconds, respectively. From the graphs, one can see that for the lower values of \(1/(1-Y)\) or \(Y\), Webster’s results fit well with the HCM 2000 results. However, for the higher values of \(1/(1-Y)\) corresponding to the data points with the LOS D or worse, a new better linear regression model was applied for each total lost time case. From these linear regression equations, the slope of these equations changes almost linearly with increasing total lost time, but the intercepts are quite similar for different lost time cases. Therefore, the following modified Webster’s model was proposed:

\[
c_0 = \frac{a + bL}{1 - Y} + c \quad (10)
\]

The parameters \(a\) and \(b\) can be obtained by linear regression on \(slope = a + bL\) versus the total lost time, \(L\). The regression results are shown in Figure 8. The parameter \(c\) is equal to the mean value of the intercepts in Figures 3 to 7. The modified Webster’s model for LOS D or worse situation is shown in Equation 11 for a four-phase intersection:

\[
c_0 = \frac{0.6L + 2.9}{1 - Y} + 40 \quad (11)
\]

The modified Webster’s model is a two-piece model, which does not have a smooth transition around the connection point of two models, Equation 1 and Equation 11. To overcome this shortcoming, the third model, the exponential type of nonlinear regression model, was proposed:

\[
c_0 = \alpha L e^{βY} \quad (12)
\]
where $\alpha$ and $\beta$ are two regression parameters. The $\alpha$ and $\beta$ were calibrated as 1.5 and 1.8 from the experimental data, respectively. Thus, the Exponential cycle length model shown as Equation 13 was obtained.

$$c_0 = 1.5Le^{1.8Y}$$

To further compare the Webster optimal minimum delay cycle length equation with the three new models, the R-squared values (coefficients of determination), $R^2$, for the above models were calculated using Equation 14 (9):

$$R^2 = \frac{SS_R}{SS_T} = 1 - \frac{SS_E}{SS_T}$$

where $SS_R$ = the regression sum of squares; $SS_E$ = the error sum of squares; and $SS_T$ = total corrected sum of squares.

Based on the R-squared results for the models shown in Table 7, the recalibrated Webster model is better than the Webster equation and the modified Webster model is the best. Figure 9 illustrated the better fittings of the modified Webster model and the Exponential cycle length model than the original Webster minimum delay cycle length equation for the case of the total lost time of 16 seconds.

**APPLICATION OF THE NEW MINIMUM DELAY CYCLE LENGTH EQUATIONS**

The modified Webster’s model and the Exponential cycle length model could be adopted in HCS-types of software, which use HCM delay method but do not have a cycle length optimization engine, to obtain a good initial estimate on the optimal cycle length once the volume, geometry, and lost times are given for an intersection.

The modified Webster’s optimal cycle length model and the Exponential cycle length model are useful to obtain the optimal signal timing corresponding to the minimum delay of the intersection. In this type of application, the sum of critical lane group flow ratios ($Y$) and total lost time ($L$) are first calculated based on traffic volumes, saturation flow rates, and signal phasing schemes. Equation 11 or Equation 13 can be used to calculate the optimal cycle length. Then the intersection’s critical degree of saturation can be calculated based on the optimal cycle length. If the LOS of the intersection is C or better, Equation 1 should be used to calculate the optimal cycle length for the case of using the modified Webster’s optimal cycle length model. Finally, the effective greens are calculated to provide equal $X$’s (degree of saturation) and proportional $y$’s (critical lane group flow ratio).

The modified optimal cycle length equation can also be used in intersection design and planning analysis. In planning or design applications, people want to design the number of lanes that an intersection needed to handle the forecasted or projected volume at some future year at a desired Level of Service corresponding to the minimum delay cycle length.

**CONCLUSIONS**

In this paper, the minimum delay cycle lengths for a wide range of different traffic and lost time situations were computed using HCS software based on HCM 2000. After comparing and modifying Webster’s minimum delay cycle length equation, the paper reached the following conclusions:
1. The first delay term in the HCM 2000 delay model is based on the first term of the Webster’s original delay equation. Until v/c ratio is equal to 1, the first term of HCM is the same as Webster’s first term delay equation. However, the second term of the delay model in HCM 2000 is different from the Webster’s original second term of delay equation. When the degree of saturation is approaching one, the delay based on Webster’s delay will become infinity, which is unrealistic. The HCM 2000’s delay model is time-dependent, thus can handle the random failure and short-term oversaturated situations.

2. Because the delay calculations are different for the HCM 2000 method and Webster’s method, the Webster’s optimal cycle length equation should be modified accordingly. Based on our experimental results, at the low traffic volume conditions, for LOS C or better, Webster’s optimal equation is still a good estimation. However, for high traffic volume conditions, the modified Webster’s optimal equation developed in this study showed better results. In addition, an exponential type regression model was developed in this paper. The Exponential cycle length model fits both high volume and low volume situations.

3. The modified Webster’s optimal cycle length model and the Exponential cycle length model are useful in signal timing design and analysis. The models can be adopted in HCS-type software as an optimization tool to provide initial estimate on optimal cycle length.

4. This study is limited to the four-phase intersection’s optimal cycle length analysis. Further studies should be conducted on two, three and other multiphase situations to develop a more generalized model. In addition, the analysis duration T is limited as 15 minutes. The effect of T should also be included in the generalized model. Nevertheless, similar research methodology as proposed in this study could be applied. Due to the lack of computing power in the 1950’s, the generality of Webster’s model has never been established either.
REFERENCES

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### TABLE 1 Level of Service Criteria for Signalized Intersection

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<th>Level of Service (LOS)</th>
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<th>Total Volumes, vph</th>
<th>Intersection Avg. Control Delay, sec</th>
<th>Xint</th>
<th>Y</th>
<th>Optimal Cycle Lengths from HCS2000, sec</th>
<th>Webster's c0, sec</th>
<th>1/(1-Y)</th>
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<td>70</td>
<td>76</td>
<td>2.63</td>
</tr>
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TABLE 5 Results for Total Lost Time of 18 Seconds

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<th>Total Volumes, vph</th>
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<th>Y</th>
<th>Optimal Cycle Lengths from HCS2000, sec</th>
<th>Webster's ( c_0 ), sec</th>
<th>1/(1-Y)</th>
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<tbody>
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### TABLE 6 Results for Total Lost Time of 20 Seconds

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<th>Total Volumes, vph</th>
<th>Intersection Avg. Control Delay, sec</th>
<th>Xint</th>
<th>Y</th>
<th>Optimal Cycle Lengths from HCS2000, sec</th>
<th>Webster's $c_0$, sec</th>
<th>$1/(1-Y)$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>24.5</td>
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<td>0.32</td>
<td>54</td>
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<td>4.17</td>
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### TABLE 7 Calculated R-squared Values for the Minimum Delay Cycle Length Models

<table>
<thead>
<tr>
<th></th>
<th>Webster Equation</th>
<th>Recalibrated Webster Model</th>
<th>Modified Webster Model</th>
<th>Exponential Cycle Length Model</th>
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<tbody>
<tr>
<td>$S_{ST}$</td>
<td>19196</td>
<td>19196</td>
<td>19196</td>
<td>19196</td>
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<tr>
<td>$S_{SE}$</td>
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<td>7620</td>
<td>824</td>
<td>2011</td>
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<tr>
<td>$R^2$</td>
<td>0.013</td>
<td>0.603</td>
<td>0.957</td>
<td>0.895</td>
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</tbody>
</table>
FIGURE 1 The Delay Illustration for HCM 2000 Method and Webster’s Method
FIGURE 2 Different Effects on Delay of Variation of the Cycle Length by Webster and HCM 2000 Method
FIGURE 3 Optimal Cycle Lengths Versus $1/(1-Y)$ for the Total Lost Time of 12 Seconds

$$y = 10.148x + 37.672$$
$$R^2 = 0.8951$$
FIGURE 4 Optimal Cycle Lengths Versus $1/(1-Y)$ for the Total Lost Time of 14 Seconds
\[ y = 13.133x + 36.844 \]

\[ R^2 = 0.9338 \]

**FIGURE 5** Optimal Cycle Lengths Versus \( 1/(1-Y) \) for the Total Lost Time of 16 Seconds
FIGURE 6 Optimal Cycle Lengths Versus $1/(1-Y)$ for the Total Lost Time of 18 Seconds
 FIGURE 7 Optimal Cycle Lengths Versus $1/(1-Y)$ for the Total Lost Time of 20 Seconds
FIGURE 8 Linear Regression of \( a + bL \) versus Total Lost Time \( L \)

\[
y = 0.5943x + 2.869 \\
R^2 = 0.9318
\]
FIGURE 9 Comparison of Webster Minimum Delay Cycle Length Equation with Three New Models