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Example Analysis and Handling of Uncertainty in the *Highway Capacity Manual* with Consideration of Traffic Diversion

Andrzej P. Tarko and Zong Tian

Dealing with uncertainty is inevitable when the performance of transportation systems is estimated and predicted. Various sources of uncertainty and how uncertainty can be handled are demonstrated. An example single stream at an unsignalized intersection is considered as an illustration. The data collected in the field were used to present sources of uncertainty and their contribution to prediction error. Interval values are proposed to represent the prediction uncertainty. This approach is advantageous over rigorous statistical estimation when distributions of the variables and interactions between them are not well specified. Also addressed is drivers' adjusting behavior to changing traffic performance. A simple volume elasticity approach is used to address traffic diversion and to provide realistic estimation of interval values of future delay. This study should be a useful introduction to the uncertainty issue in traffic analysis based on the *Highway Capacity Manual.*

Uncertainty about future traffic and its performance is inevitable. There is a growing interest among researchers to quantify and handle uncertainty in traffic analysis (*1*–*3*). Although handling and reporting uncertainty in transportation studies requires caution, these techniques are welcomed by the majority of *Highway Capacity Manual* (*4*) (HCM) users and can promote more beneficial decisions (*5*,).

It is not easy to predict the number of vehicles that will pass a given spot in the next 15 min. If the future mean is known, the predicted number of vehicles cannot be stated as a single number but through a distribution. The situation is worse when the mean value is also uncertain and it has to be expressed through its own distribution. In such a case, uncertainty is affected by unexplained variability of individual counts around the mean value and by the uncertain value of the mean.

It may be said that if the mean value is to be predicted, the prediction error is lower and the prediction more certain than for individual values. In the traffic counts mentioned earlier, the variability of individual counts does not affect the ability to predict the mean count unless this variability does not allow for accurate estimation of the recent mean. Fortunately, the HCM applications deal with mean values (*4*). After all, public perception of traffic at some road locations is based on multiple experiences of many people. Although the mechanism for formation of public opinion about some road facility is not well understood, consideration of mean traffic conditions seems to be more justified than consideration of particular traffic conditions that occur once.

There are three primary sources of uncertainty in the results generated by the HCM models (and any other models):

1. Model errors that occur even when all the inputs are correct,
2. Errors caused by incorrect model parameters (same values for multiple locations), and
3. Errors caused by incorrect volume and other inputs.

Selected sources of prediction errors are discussed here, and one method of dealing with uncertainty out of many possible methods, namely, interval analysis, is presented. Interval analysis uses interval values determined by their lowest and highest limits. An interval value (sometimes called interval of possibilities) can be compared with a confidence interval with a high confidence level. However, statistical properties of the variable are not considered, which makes this method simple and practical.

A stream of vehicles turning left from a road controlled by a stop sign will be discussed in detail to illustrate a variety of sources of uncertainty. This discussion is to illustrate the topic of uncertainty in traffic predictions, not to improve any of the existing traffic models, in order to increase the appreciation of any effort aimed at handling uncertainty in traffic predictions.

DELAY MODEL

Traffic delays experienced by motorists depend on the traffic demand approaching a roadway facility and the ability of that facility to carry the traffic (capacity). The HCM delay equation for unsignalized streams includes these two inputs and nothing else. In this section the field measurements, evaluation of the existing HCM delay model, and a model modified on the basis of the evaluation results are discussed.

Field Measurements

Traffic at the intersection of San Antonio Road and the Highway 101 exit in Mountain View, California, was videotaped on three different days during the weekday p.m. peak periods, between 3:00 and 5:00 p.m., for a total of almost 5 h. The intersection layout is shown in Figure 1. This intersection has only through traffic on the major street (San Antonio Road). Only the stop-controlled left-turn movement on the minor street was studied. The arrival times (joining the queue), the time of moving to the first stop-line position, and the departure times of vehicles turning left from the Highway 101 exit

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A. P. Tarko, Civil Engineering Department, Purdue University, 550 Stadium Mall Drive, West Lafayette, IN 47907. Z. Tian, Texas Transportation Institute, Texas A&M University System, College Station, TX 77843.

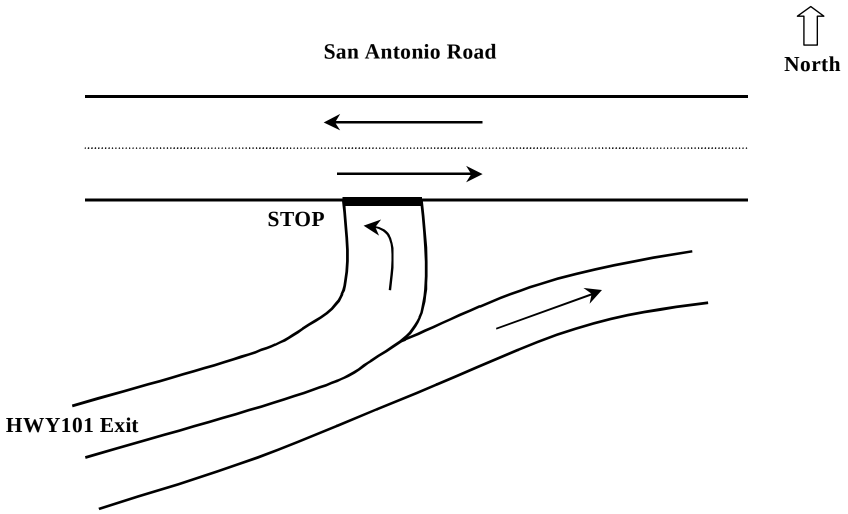

FIGURE 1 Site map.

were collected manually from the videotapes. These data allowed the estimation of the following quantities for 15-min intervals: capacity, arrival rate, average delay, and initial queues at the interval onset. These and other intermediate results are shown in Table 1.

In addition to the parameters shown in Table 1, the following parameters were directly obtained from the database: the service time, defined as the time that a minor-stream vehicle stays at the stop-line position; the move-up time, defined as the time it takes for a vehicle to move from the second-in-queue position to the stop-line position; and the follow-up time. These are important parameters for capacity estimation in the field. The delay values are average delays of vehicles that joined the queue during the interval. Some of the vehicles arriving in the interval departed in the next interval.

The capacity values shown in Table 1 were estimated using the same methodology documented in the NCHRP Project 3-46 Final Report (*6*). The capacity estimation model is reentered as

$$c = \frac{3600}{t_s + t_{mv}} \tag{1}$$

where

c = capacity for minor-stream movement (vph),
t_s = average service time (s), and
t_{mv} = average move-up time (s).

The above capacity estimation model proved to be valid under both saturated and undersaturated conditions (*6*).

It should be noted that delay estimates in the field are less reliable, particularly when the queue is long. Accurate arrival time of vehicles can only be observed when the queue is about 10 vehicles long or shorter. At this location, the queues on the minor-street approach sometimes exceeded 10 vehicles. In this case, field delays could be either slightly overestimated or underestimated.

TABLE 1 Volume and Delay Measurements for Study Site

Interval No.	Major Street Volume (veh/h)	Left Turn Volume (veh/h)	Measured Capacity (veh/h)	Average Delay	Initial Queue	v/c
1	1132	344	357	83.0	10	0.96
2	1164	348	363	73.3	8	0.96
3	1140	308	328	48.6	6	0.94
4	932	320	419	22.5	4	0.76
5	1032	368	355	53.2	1	1.04
6	860	408	442	45.4	1	0.92
7	552	456	554	28.7	0	0.82
8	796	396	421	37.3	0	0.94
9	712	460	461	45.5	7	1.00
10	888	392	432	33.3	3	0.91
11	804	388	446	35.5	0	0.87
12	756	376	486	22.1	5	0.77
13	756	336	451	17.9	5	0.75
14	980	388	405	34.8	4	0.96
15	996	396	359	63.8	0	1.10
16	1108	384	380	93.4	12	1.01
17	1000	400	415	90.1	8	0.96
18	968	440	436	86.6	10	1.01
19	1008	472	460	79.6	8	1.03
20	904	392	405	95.7	11	0.97

HCM Delay Model

A time-dependent version of the M/M/1 equation is used in the HCM to predict the average delay:

$$d = \frac{3600}{c} + 900 \cdot T \cdot \left[v/c - 1 + \sqrt{(v/c - 1)^2 + \frac{8 \cdot v/c}{c \cdot T}} \right] + 5 \tag{2}$$

where T is the analysis time period, here 0.25 h.

Figure 2 compares the measured delay with the delay estimates calculated with Equation 2. It appears that the HCM delay model overestimates delays lower than 75 s. This discrepancy can be explained by lower randomness of traffic observed in the field than that assumed in the model. It is not a surprise since the measured delays apply to known (read “fixed”) volumes and capacities, whereas the predicted delays apply to average traffic. Thus, the additional variability is added to the predicted delays through volume and capacity varying around their means. The second discrepancy is for delay values above 75 s when the predicted delays are significantly lower than the measured delays. This discrepancy indicates that a significant delay factor for congested conditions is missing in the model, probably the initial queues.

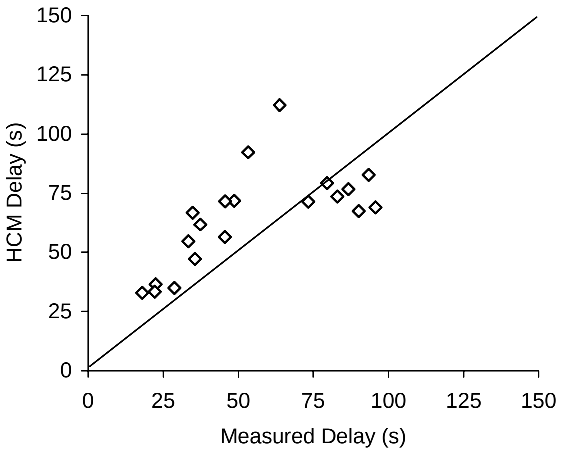

FIGURE 2 Measured delay versus HCM delay estimates.

Modified Model

To take care of the two weaknesses, Equation 2 has been modified as follows:

1. The value 8 was replaced with the expression $(4 - 2Q_b/3)$, where Q_b stands for the initial queue in vehicles. The expression $(4 - 2Q_b/3)$ was obtained by fitting the delay equation to the field data. The expression returns values lower than 8 and represents lower traffic randomness.
2. A third term, d_3, was added, estimating the additional delay caused by an initial queue. The expressions published in the HCM for signalized intersections were used.

The new equation is as follows:

$$d = \frac{3600}{c} + 900 \cdot T \cdot \left[v/c - 1 + \sqrt{(v/c - 1)^2 + \frac{\max(0, 4 - 2 \cdot Q_b/3) \cdot v/c}{c \cdot T}} \right] + 5 + d_3 \quad (3)$$

where d_3 is unmet demand delay (effect of initial queue).

Figure 3 compares the delays measured in the field and the delays estimated from Equation 3. The spread of points around the regression line may be caused by the measurement error and by variability of delay. The spread of points is growing with the average delay (heteroscedasticity). The maximum likelihood–based fitting of the model to the data points indicated that the standard deviation of the points from the regression line is 24% of the mean value estimated with Equation 3. In other words, in 95% of cases the actual value is somewhere between $0.5d$ and $1.5d$ (or $d \pm t\,0.24d$), where t is the Student's statistic value for 95% confidence and model degrees of freedom and d is the value calculated with Equation 3. Such strong delay variability in short intervals for known volume, capacity, and initial queue is possible at urban intersections where vehicles form platoons. High average delay occurs when the platoons on the minor and major approaches arrive simultaneously, whereas low delay occurs when the platoons "miss" each other.

It should be noted that the $0.5d$–$1.5d$ intervals apply to individual 15-min observations while the mean values are of interest. The variability range applicable to the mean values is narrower. For the purpose of further analysis, the confidence interval for the mean is estimated as $0.85d$–$1.15d$ (or $d \pm t \cdot 0.24d/\sqrt{f}$), where t and d are as defined in the previous paragraph and f is the degrees of freedom of the delay model. It must be stressed that this variability is caused solely by the deficiencies of the model and does not include the effect of uncertain input since the volume and capacity were assumed to be perfectly known.

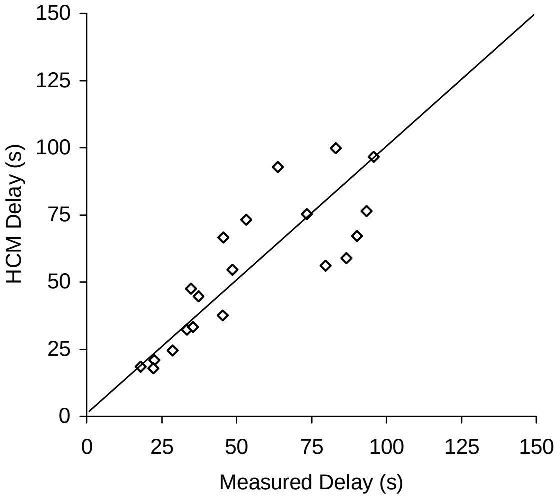

FIGURE 3 Measured delay versus delay estimated with modified HCM equation.

CAPACITY PARAMETERS

The previously discussed delay prediction was based on known volume and capacity. The situation dramatically changes when volume and capacity are to be predicted as a part of the delay prediction. Typically, capacity is calculated using default values of capacity parameters: critical gaps and follow-up times are used, since their observation is not easy.

"Local" values of critical gap and follow-up time for the example flow were estimated from the field data. The average observed values are $t_c = 5.2$ s and $t_f = 3.6$ s. Figure 4 shows the capacity curves calculated with the HCM equation and using the obtained local parameters (*upper curve*). The observed capacity values (*open squares*) are also shown. As can be seen, the HCM capacity curve with the local parameters fits the field-measured capacity well. Some inconsistencies exist and are caused partly by unexplained variability of

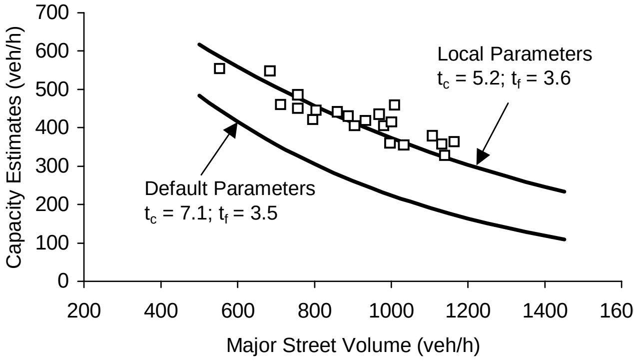

FIGURE 4 Local capacity parameters and local capacity.

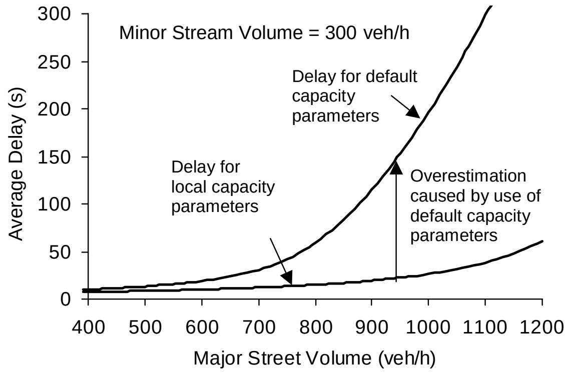

FIGURE 5 Bias caused by using default capacity parameters instead of local values.

capacity and partly by the decreasing value of the critical gap for long delays (*7*). For comparison, the capacity curve for default parameter values ($t_c = 7.1$ s, $t_f = 3.5$) recommended by the HCM is also shown. The capacity underestimation is obvious. This capacity estimation bias transfers to a strong overestimation of delays (Figure 5).

It should be noted that such bias caused by local parameters is unknown if the local parameters are unknown. In such a case, the parameters can be represented by their distribution or by reasonable ranges of values (interval values). The potential range of capacity predictions could then be estimated further through a range of possible mean delays. This is the way uncertainty about true capacity parameters will be handled in this paper.

This part of the discussion should be closed with a methodological conclusion. Significant sensitivity of models to their parameters should prompt model developers to provide sufficient information about parameters' variability to help HCM users in handling uncertainty in a reasonable manner. If the impact of uncertain parameters is too large, it should warrant local studies of the parameters by the HCM users. On the other hand, if local studies were prohibitively expensive or complex, research would be justified to improve the models by providing a method of predicting the parameter values for local conditions in lieu of fixed default values.

TRAFFIC VOLUMES

Predicting traffic volumes, particularly for a remote design period, is a challenging task. Variability of traffic, uncertainty about future land development, and travelers' behavior contribute to this difficulty. Most sources of uncertainty about future volume at a certain location do not pertain to that location since the roadway network is a system of connected links, and changes in one part of the network may considerably influence volumes somewhere else. Transportation planners approach this problem by predicting network volumes for various scenarios. A range of possible future volumes is obtained from this exercise.

It is assumed, for the purpose of further discussion, that multiple planning scenarios of the highway network, including the considered example of left-turn flow, have returned a range of possible flows for a typical afternoon peak hour of a target year (Table 2). The difference in the results stems from assumed various levels of traffic generation by a new development. The last column of Table 2 gives the average delay predictions from Equation 3 (modified HCM delay model with assumed zero initial queue and, consequently, $d_3 = 0$).

DELAY PREDICTION

Delay Prediction Without Traffic Diversion

The delays shown in Table 2 reflect the variability of input volume, but they do not include the effect of uncertainty associated with the model and capacity parameters as presented in the previous sections. One may attempt to incorporate these additional uncertainties by recalculating delay for various possible capacity parameters and by multiplying the obtained delay values with model factors 0.85 and 1.15 to incorporate the model-inflicted uncertainty of the mean delay.

It is assumed that values of the critical gap vary between 4.9 and 5.5 s, and the follow-up time is between 3.4 and 3.8 s. Example results of this approach are given in Table 3. Only combinations of

TABLE 2 Ranges of Volume for Example Site

Scenario	Major Street Volume (veh/h)	Left-turn Volume (veh/h)	Average Delay (s)
Low demand	1000	255	24.4
High demand	1150	289	47.4

TABLE 3 Delay Predictions Without Traffic Diversion

Delay Interval Limits	Major Volume (veh/h)	Left-turn Volume (veh/h)	Critical Gap (s)	Follow-up Time (s)	Model Factor	Delay Value (s)
Bottom	1000	255	4.9	3.4	0.85	17.0
Upper	1150	289	5.5	3.8	1.15	89.2

input values that produce the lowest and the highest delays are presented, representing two extreme conditions that can be encountered. The approach adopted here is rather crude since it does not include dependency between some of the inputs and parameters. For example, the critical gap tends to be lower while the delay is growing (higher major volume). Such dependencies, if known, can be reflected when combinations of input and parameter values are generated.

The delay interval is quite wide. More important, the upper limit is so high that it is likely to be unacceptable to drivers. Some diversion of traffic to less-congested routes may be expected, and the obtained high delay may not be possible. Traffic generation and diversion induced by changes in the infrastructure are well known and have been described (*8*). The phenomenon of drivers adjusting to traffic conditions through diversion should not be overlooked when the reasonable range of possible delays is determined.

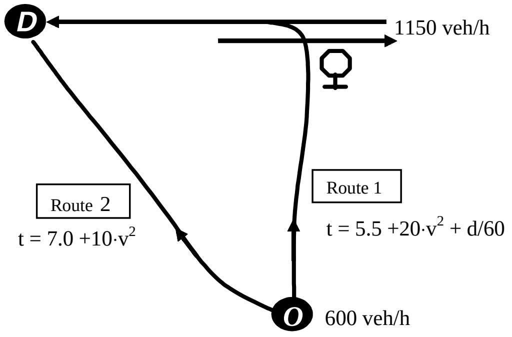

FIGURE 6 Example single OD pair network (*v* = flow along route in thousands of vehicles per hour, *d* = delay of left turn in seconds, *t* = travel time between *O* and *D* in minutes).

Volume Function

In the current HCM procedures, volume is a fixed input. If this assumption is relaxed and it is postulated that traffic volume can change in response to various delay values, such a relationship is called a volume function. To illustrate a volume function, a single origin–destination (OD) network is shown in Figure 6. In the high-demand scenario, a total number of 600 vehicles travel during the design hour to the destination using Routes 1 and 2. Route 1 includes the considered left turn. The generalized OD travel time in minutes is $t = 5.5 + 20v^2 + d/60$, where v is the flow using this route in thousands of vehicles per hour and d is the average delay of the left turn in seconds. Route 2 represents all other alternative routes that vehicles can take to divert from Route 1. Its travel time in minutes is $t = 7.0 + 10v^2$. This generalized route is longer than Route 1, and thus the free-flow travel time of 7 min is longer than it is along Route 1, but the travel time growth with volume increase is slower. The total OD flow splits between the two routes in a way that equalizes the travel times along both routes. For 600 vehicles traveling between the origin and the destination, 289 vehicles use Route 1 and 311 vehicles use Route 2. The travel time is 7.97 min (the same along both routes) and the delay of the left-turn flow is 47.4 s.

Because of some unknown factors other than demand, the delay of the left-turn stream can be different from the predicted delay: for example, major-street volume may be different, critical gap may be shorter. Table 4 shows assumed various sets of inputs different from the original input values. The model multiplier in the fourth column incorporates changes in the delay caused by other factors not included in the input, for example, the effect of adjacent signalized intersections. The left-turn volume and the delays were recalculated for the new data inputs and are presented in Table 4 and Figure 7. The obtained relationship represents the volume-adjusting abilities to the changes in the delay conditions. This near-linear relationship is the volume function mentioned earlier. As could be expected, the higher delays caused some drivers to divert, and thus the left-turn volume drops. The volume curve slope is –0.97, or, in other words, a 1-s increase in delay caused a 1.0-veh/h reduction in volume. Similar analysis for the low-demand scenario (400 veh/h OD flow) indicated that the volume elasticity was 1.3 veh/h per second. It must be stressed that the volume functions were obtained only for illustrative purposes and that actual volume functions may vary from case to case.

TABLE 4 Delay Model Inputs and Resulting Volume and Delay for High-Demand Scenario

Input Set	Major Volume	Critical Gap	Model Multiplier	Route 1 Volume	Left Turn Delay
0	1150	5.0	1.0	296	40.3
1	1000	5.0	1.0	307	28.2
2	1300	5.0	1.0	279	58.4
3	1150	4.0	1.0	315	20.1
4	1150	6.0	1.0	254	84.1
5	1150	5.0	0.5	312	23.4
6	1150	5.0	2.0	272	66.1
7	1000	4.0	0.5	325	8.7
9	1300	6.0	2.0	193	144.6

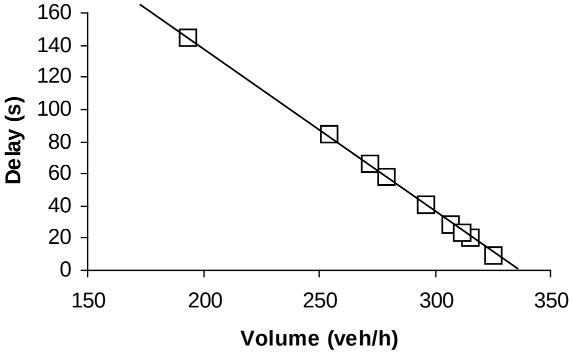

FIGURE 7 Volume function for the high-demand scenario.

Delay Prediction with Traffic Diversion

Two functions are needed to predict the volume and the delay:

1. Delay function of volume (supply function), and
2. Volume function of delay (demand function).

The HCM provides the delay function, whereas the volume function has to be derived from the network modeling results or from a simplified analysis of opportunity for diversion. Table 3 has been amended with the additional information about volume elasticity. The extended input set is shown in Table 5. The last three columns define the volume function for each scenario. The volume function takes the form of a straight line with the slope equal to the corresponding elasticity and passing through a point representing the left-turn volume and the corresponding average delay.

Figure 8 explains the way the correct upper limit of the delay interval can be calculated for the high-demand scenario using the input data in Table 5. Curve A is the average delay without uncertainty consideration. Point 1 on this curve is the prediction of delay (47.4 s) for the corresponding volume of 289 veh/h given in Table 2. Curve B is the upper limit of the delay curve obtained by assuming a long critical gap and long follow-up time and multiplying the obtained delay by a model factor of 1.15. The corresponding estimated delay, 89.2 s, for the volume of 289 veh/h is marked as Point 2 (see Table 3). Curve C is the volume curve constructed by using information given for the high-demand scenario in Table 5. A straight line with a slope –0.97 has been drawn through the point (volume = 289 veh/h, delay = 47.4 s). The intersection point of Curve C with Curve B gives Point 3 (volume = 268 veh/h, delay = 68.5 s), which is the sought solution and correct prediction of the upper-limit delay and the corresponding volume. The same technique was used to obtain the bottom limit of delay for the low-demand scenario. Table 6 summarizes the final delay predictions. The range of possible delays is considerably narrower than the range obtained without traffic diversion consideration (Table 3).

The solution presented graphically in Figure 8 can easily be obtained by solving the following equation for delay:

$$d = f[v(d)] \tag{4}$$

where $f(v)$ is the HCM delay model and $v(d)$ is the function of the volume as

$$v = v_0 + e \cdot (d - d_0) \tag{5}$$

where v_0 is mean volume, d_0 is mean delay, and e is volume elasticity.

CONCLUSIONS

Various sources of uncertainty in delay predictions caused by model imperfection, variability of capacity parameters, and variability of volume have been illustrated. The uncertainty was represented through interval values. This simplified method of representing uncertainty considers neither distribution of variables nor their covariance. It is, however, practically attainable.

The method produces the best and the worst scenarios that represent possible ranges of traffic conditions. An optional approach could be proposed in which the average conditions are considered a primary solution, and the worst scenario is checked for the uncertainty effect.

The strong effect of capacity parameters was demonstrated for the case in which the volume-to-capacity ratio is close to 1. In such cases, allowing for variability of capacity parameters, together with other variability, may yield delays too high to be reasonable. Drivers would simply divert to other routes to avoid the congested road. Diversion of traffic is an important part of uncertainty handling, and it should be considered to obtain valid results. Drivers' diversion mitigates the effect of uncertainty and narrows the delay interval produced.

A simple way of representing the diversion behavior of drivers is volume elasticity, defined here as a slope in the linear volume-delay relationship. The volume elasticity tells how much volume diverts (or arrives) with a unit change in delay. The use of the HCM delay function and the volume functions as determined by elasticity is proposed to find valid delay interval values that take into account the diversion behavior of drivers.

To be able to apply the proposed approach, more research is needed, including the following topics:

1. Variability of the field values around HCM-generated results should be quantified to allow incorporating the effect of model imperfection into the assessment of results uncertainty. This exercise is sometimes called model validation.

TABLE 5 Input Data for Predicting Delay with Traffic Diversion

Scenario	Major Street Volume (veh/h)	Left-turn Volume (veh/h)	Average Delay (s)	Volume Elasticity (veh/h/s)
Low volume	1000	255	24.4	1.3
High volume	1150	289	47.4	1.0

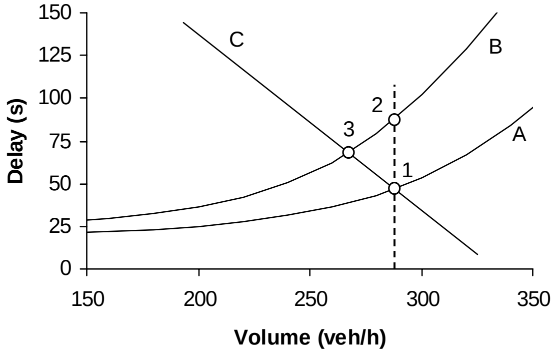

FIGURE 8 Predicting delay when traffic diversion is considered.

2. Sensitivity of the HCM models to input and parameters should be investigated to determine what input and parameters have the strongest impact on the results quality.

3. Investigation of intersite variability of most "influential" parameters is needed to check a potential impact on the results. The parameters that influence the results strongly and are difficult to estimate by HCM users should be associated with methods of their prediction more elaborated than default values. Equations for critical gap and follow-up time prediction are good examples.

4. Further thought should be given to the concept of volume elasticity for estimating the effect of volume inaccuracy on the HCM-generated results. The idea of default values of volume elasticity should be checked. It is interesting to consider whether a set of default values could be proposed as a more practical option than requesting case-specific values for each analysis.

5. The interval-based approach of handling uncertainty in calculations should be tested for more complex models to check its usefulness.

TABLE 6 Volume and Delay Predictions with Traffic Diversion

Delay Limit	Left-turn Volume (veh/h)	Delay Value (s)
Bottom	264	17.5
Upper	268	68.5

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