Effectiveness of Lead–Lag Phasing on Progression Bandwidth

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Quantitative assessments are provided on two signal timing issues related to progression bandwidth maximization: the effectiveness of using lead–lag phasing and the effect of the number of signals on progression bandwidth. A computer program was developed to generate multiple signal system scenarios randomly and to provide maximum bandwidth solutions. The randomly generated signal system scenarios represented various signal systems likely to be seen in the real world. On the basis of these randomly generated system scenarios and their associated maximum bandwidth solutions, conclusions were drawn regarding the two issues. Lead–lag phasing had a significant advantage over the leading-left-turn and lagging-left-turn phasing schemes to provide maximum bandwidth solutions. At any signal in a system, lead–lag phasing was used in more than 70% of the cases compared, with about 20% for leading-left-turn phasing and 10% for lagging-left-turn phasing. The number of signals had a profound impact on attainability and bandwidth with an increasing number of signals in a system.

Maximizing progression bandwidth is a primary objective when developing coordinated signal timing plans (1, 2). A larger progression bandwidth implies that more traffic on an arterial can progress through the signals without stops. Furthermore, bandwidth-based signal timing is preferred because it meets drivers’ expectations. Besides general signal timing parameters of cycle length and phase split, phasing sequence is perhaps the most important element when maximizing progression bandwidth is desired (3, 4).

Phasing sequence refers to the order of left-turn and through phases on the main street, which include dual-leading left turn (lead), dual-lagging left turn (lag), and lead–lag (Figure 1). Although the effectiveness of using lead–lag phasing has been recognized by scholars and engineers for maximizing progression bandwidth, no quantitative assessment has been made on when lead–lag phasing can improve bandwidth over other phasing sequences, or how much bandwidth improvement can be achieved. A related issue is how bandwidth is affected by the number of signals in a system.

The purpose of this study is to provide such quantitative evaluations. Its primary objective is to answer two specific questions when seeking a maximum bandwidth solution for a signalized arterial:

- How often is a lead–lag phasing needed at an intersection?
- How will maximum bandwidth change with an increased number of signals in a system?

To answer these questions, a computer program was developed that can randomly generate hundreds of network cases and provide the maximum bandwidth solutions for each. The cases were generated from specific ranges of variable and parameter values, representing various signal system configurations likely to be found in the real world. With the results of analyzing many such cases, sufficient data and valid statistics were gathered to draw definitive conclusions. The study assumes protected-only left-turn controls on the arterial street; therefore, lead–lag phasing can be used without the yellow-trap conditions typically seen with standard protected-permitted left-turn controls. Although it is recognized in practice that having an entire arterial with protected-only left-turn controls is rare, this study mainly addresses how effective a lead–lag phasing would be, if the phasing were available at a given intersection.

The paper is organized as follows. First, a historical view of signal timing and related literature is given. Then, the development of the computer program is discussed, and the computer program results are analyzed. Finally, conclusions drawn from the study are presented.

BACKGROUND

Pioneering research on signal timing optimization started with the development of mathematical models and associated computational algorithms. The first effort on this topic involved a computer program by Brooks to maximize progression bandwidth given two-phase traffic signals along an arterial (5). Following Brooks’s concept, Morgan and Little formulated a mathematical model for the bandwidth maximization problem (6, 7). Their model used mixed-integer programming, from which an equal maximum bandwidth could be obtained for both directions. Since the publication of that work, many studies have followed in this area to improve modeling theories and techniques. The first major enhancement was the capability of handling multiphase traffic signals (8–11), which led to the development of several popular computer software packages for signal timing optimization, including PASSER II (2), MAXBAND (9), and MULTIBAND (12, 13). Efforts to improve bandwidth-based optimization models and their computing algorithms continued up to the late 1990s (e.g., 14–17).

Work by Messer et al. established the primary computing algorithm for the PASSER II and MAXBAND software packages (8). Progression bandwidth is maximized by minimizing a combined lower interference (I_1) and upper interference (I_2). The relationship between bandwidth and interference is illustrated in Figure 2 for a three-signal system of two-phase signals (i.e., no left-turn phases). The
one-direction bandwidth \((B)\) is obtained by subtracting \(I_t\) and \(I_u\) from the minimum arterial green \((G_{min})\).

For signals without left-turn phases, the lower and upper interferences for each intersection are determined by locating the signal offset at half-cycle increments, a technique that has been referred to as the half-integer optimization technique \((8)\). For intersections with left-turn phases of equal length, the maximum bandwidth for both directions is obtained through a mixed-integer programming optimization process. Messer et al. proposed an algorithm for maximizing the sum of progression bandwidth for both directions by first obtaining the maximum bandwidth for the outbound direction, then minimizing the interference for the inbound direction \((8)\). The outbound direction can be designated as the direction that has the higher minimum arterial green \((i.e., G_{min} \geq G_{in})\). The total maximum bandwidth for both directions \((B_{max}\) in seconds) can be obtained by minimizing the inbound interference \((I_{in})\) with the use of the following equations:

\[
B_{max} = G_{in} + G_{min} - I_{in} \tag{1}
\]

\[
I_{in} = \min\left[\max(I_{t,j}) + \max(I_{u,j})\right] \quad j \in 1 \sim m \quad j \not= x \tag{2}
\]

\[
I_{u,j} = G_{in} - (T_{x,j} + T_{y,j} - \gamma_{x,j} + \gamma_{y,j} + G_{in}) \tag{3}
\]

\[
I_{L,j} = T_{x,j} + T_{y,j} - \gamma_{x,j} + \gamma_{y,j} + G_{in} \tag{4}
\]

\[
0 \leq I_{u,j} < C \tag{5}
\]

\[
0 \leq I_{L,j} < C \tag{6}
\]

where

\[
x = \text{intersection that has the minimum inbound green;}
\]

\[
m = \text{number of signals in the system;}
\]

\[
I_{u,j} \text{ and } I_{L,j} = \text{upper and lower interferences, respectively, caused by phasing sequence } p \text{ of intersection } j;
\]

\[
G_{in} \text{ and } G_{min} = \text{inbound and outbound green time, respectively, at intersection } x \text{ (i.e., } G_{in} = G_{min};
\]

\[
G_{in} \text{ and } G_{min} = \text{inbound and outbound green time, respectively, at intersection } j;
\]

\[
T_{x,j} \text{ and } T_{y,j} = \text{travel times between intersections } x \text{ and } j \text{ in both directions;}
\]

\[
\gamma_{x,n} = \text{relative offset of } G_{in} \text{ with respect to } G_{min} \text{ for phase sequence } n;
\]

\[
\gamma_{y,p} = \text{relative offset of } G_{in} \text{ with respect to } G_{min} \text{ for phase sequence } p; \text{ and}
\]

\[
C = \text{cycle length.}
\]

The above equations demonstrate that interference increases \((i.e., bandwidth decreases)\) as the number of signals in a system increases. At a certain level, bandwidth may no longer be achievable \((19)\). In assessing the effectiveness of a bandwidth solution, the measure of attainability usually is used. Attainability \((A)\), as defined in Equation 7, indicates how good a bandwidth solution is compared with the maximum possible solution for given traffic conditions and phase splits:

\[
A = \left(1 - \frac{I_{in}}{G_{min} + G_{in}}\right) \times 100 \tag{7}
\]

Theoretically, attainability has a minimum value for two-way arterials, which is

\[
A_{min} = \left(1 - \frac{G_{min}}{G_{in} + G_{min}}\right) \times 100 = \frac{G_{min}}{G_{min} + G_{in}} \times 100 \tag{8}
\]

The \(A_{min}\) value implies that a one-direction \((outbound)\) progression bandwidth always can be obtained at \(G_{min}\); therefore, the value should be no less than 50\%. Attainability increases as the bandwidth for the inbound direction increases.

**SOFTWARE DEVELOPMENT AND DATA ANALYSIS**

The authors developed a computer program to derive maximum bandwidth signal timing solutions for multiple scenarios and assist in the data analysis. The program can randomly generate multiple traffic system scenarios on the basis of specified ranges of key system elements, such as number of signals, travel times \((to represent spacing and speed between signals)\), cycle length, and phase split. For example, each run of the program can automatically generate 100 traffic system scenarios with different system characteristics. For each system scenario, the maximum bandwidth solution was derived according to the computing algorithm developed by Messer et al. \((8)\). The phasing sequence at each intersection also was recorded.
Although it can be extended to handle more intersections, the program currently is limited to analyzing five signals. Still, the most important conclusions can be drawn from the analysis.

The following assumptions were made to generate the data and conduct the analysis:

- Each intersection had exclusive protected-only left-turn phases on the arterial street, so that all the left-turn phasing sequences shown in Figure 1 can be used at all the intersections.
- No side-street phasing sequence was specified, because side-street phasing is irrelevant to arterial progression bandwidth.
- Cycle length ranged between 60 and 140 s.
- The total main street phase split (effective green) was fixed at 60% of the cycle length, and the left-turn phase split ranged between 10% and 30% of the cycle length. The through phase split was automatically calculated after the left-turn phase split was determined, which varied between 30% and 50% of the cycle length.
- Travel time between adjacent intersections ranged between 15 and 25 s, representing different distances and travel speeds. Different travel times in the two directions were allowed.

To determine which phasing sequence is most effective for maximizing progression bandwidth, 100 signal system cases were randomly generated with the above assumptions. Maximum bandwidth solutions were derived, and the phasing sequences were recorded at each intersection used to produce the bandwidth solutions. It was noted that one particular system may have several maximum bandwidth solutions with different phasing sequences. In this case, all phasing sequences that produced the same maximum bandwidth solutions were recorded. Figures 3 to 6 (representing analyses using

![Figure 3](image3.png) Frequencies of each phasing sequence for maximum bandwidth solutions: two signals.

![Figure 4](image4.png) Frequencies of each phasing sequence for maximum bandwidth solutions: three signals.
two to five signals, respectively) show the results, illustrating the frequencies in which a particular phasing was used at each intersection. Although 100 system scenarios are considered significant and the results should be representative to general cases in the field, variations from the true statistics still exist in the results. In other words, results might be slightly different if another 100 cases were to be generated (Figures 5 and 6).

Figure 3 (a two-signal system) shows that 84% of the cases involved using lead–lag phasing at both signals when the maximum bandwidth solutions were achieved. About 11% involved leading-left-turn phasing, and about 5% involve lagging-left-turn phasing. It was concluded that in most cases, lead–lag phasing was more likely (84%) to produce the maximum bandwidth. Leading-left-turn phasing (11%) normally produced better bandwidth than lagging-left-turn phasing (5%). When the two left-turn phases had the same splits, leading- and lagging-left-turn would always produce exactly the same bandwidth, because both directions had the same start and end of the arterial green. However, when the two left-turn phases did not have the same splits, leading- and lagging-left-turn phasing produced different bandwidth solutions because of the different starts of green for both directions. However, leading-left-turn phasing usually produced better results than lagging-left-turn phasing.

Figure 4 (a three-signal system) shows that the middle signal had a slightly higher frequency of using lead–lag phasing (78%) than the two end signals (73%)—that is, the middle signal was more likely to use lead–lag phasing than the other two. Similarly, leading-left-turn phasing generally produced better bandwidth results than lagging-left-turn phasing. Similar trends can be observed in Figure 5 (with four signals) and Figure 6 (with five signals).
Figure 7 shows the average frequencies of each phasing sequence for all signals when the maximum bandwidth solutions were achieved. As the number of system signals increased, the frequency of using lead–lag phasing decreased slightly, suggesting that some signals in the system could use other phasing sequences while still obtaining the maximum bandwidth solution. However, this decline seemed to level out as the number of signals exceeded five. Nevertheless, this frequency still remained high (>70%), suggesting that lead–lag phasing was still the most effective for maximizing progression bandwidth.

Figure 8 shows how bandwidth attainability changed with the number of signals in a system. As can be seen, attainability decreased nonlinearly with the increase in the number of system signals. As discussed previously, attainability indicates how close the bandwidth solution is to its maximum possible bandwidth. An attainability value of 100% indicates that the absolute maximum bandwidth has been achieved. In theory, attainability has a minimum threshold, as shown in Figure 8 and expressed in Equation 8. When this threshold is reached, there will be no feasible bandwidth for the other direction. For cases analyzed, this threshold is projected to be met when the number of signals reaches seven. Therefore, when a signal system involves many signals (e.g., more than seven), signal timing solutions focusing on only bandwidth may not be desired, and other signal timing techniques may be sought, such as system partition (18) or solutions that focus on partial progression (19, 17).

SUMMARY AND CONCLUSIONS

The paper addresses two issues related to bandwidth maximization for coordinated signal systems. A computer program was developed to randomly generate multiple signal system scenarios and to produce
maximum bandwidth solutions. On the basis of the cases analyzed, the following major conclusions can be drawn:

- Lead–lag phasing showed significant advantages over other phasing sequences in maximizing progression bandwidth. For a signal system with five or fewer signals, more than 70% of the cases used lead–lag phasing to produce a maximum bandwidth solution. This level seemed to remain constant with more signals in a system.
- In a system with more than two signals, the middle signals were more likely to use lead–lag phasing than the signals at the ends.
- When the phase lengths of the left-turn phases were the same, leading- and lagging-left-turn phasing did not show any difference in terms of maximizing progression bandwidth. However, when the lengths of the two left-turn phases were different, leading-left-turn phasing showed some advantage over lagging-left-turn phasing.
- Attainability tended to decline nonlinearly with the increase of number of signals in a system. At a certain level, bandwidth may no longer be obtainable for one direction, suggesting that signal timing based solely on bandwidth may not be practical at that level.

The above conclusions were drawn from an analysis of systems with five or fewer signals. Although the conclusions regarding the effectiveness of lead–lag phasing are not expected to change with more signals, expansion of the program to analyze more than five signals is considered valuable to verify the effect of signal number on bandwidth attainability.

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