INSTRUCTIONS: Write your solutions concisely, and in the space provided. Please show your work or otherwise justify your claims to receive full credit.

1. Let $A = [0, 2]$, $B = [-1, 1]$, and $C = [0, 1]$ be subsets of $\mathbb{R}$. Suppose random variable $X$ has density function $f_X(x) = 1/3$ if $x \in [0, 3]$, $f_X(x) = 0$ otherwise.

   a. What is $(A \cup C) \cap B^C$?

   b. Find $P(X \in B) = P(-1 \leq X \leq 1)$.

   c. Find $P(X \in C | X \in A)$.

   d. Find the $F_X(x)$, the cumulative distribution function (CDF) of $X$. 

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2. What is the number of unique rearrangements of the word SALAMANDERS?

3. How many 6-letter words can be made from a 26-letter alphabet (which has 5 vowels) if we require that it has 3 distinct vowels and 3 distinct consonants?
4. A team of 7 ornithologists, including Dr. Burtt, are trying to find a rare bird known to be in a nearby forest. They randomly divide into a team of three and a team of four, and split up to search along two trails. The team of four takes trail A, the others take trail B. Along trail A, there is a 30% chance the bird will be found, and a 20% chance it will be found along trail B (there is zero chance it will be found by both teams).

a. What is the probability that Dr. Burtt’s team finds the bird?

b. Suppose Dr. Burtt’s team finds the bird. What is the probability it was found along trail A?
5. Let $X$ be a discrete random variable with sample space $S = \{1, 2, 3, 4\}$ and $P(X = k) = ck$ for each $k \in S$.
   a. Find the value of $c$.

   b. Find the CDF of $X$.

   c. Find the $E[X]$, the expected value of $X$. 
6. State the definition of sample space.

7. State the definition of mutually exclusive for two events $A$ and $B$.

8. State the definition of independence for two events $A$ and $B$.

9. Review the following homework problems and class example:
   (a) HW problem 2.5.22 (Repeated independent events)
   (b) HW problem 3.2.11 (Binomial probability example)
   (c) HW problem 3.2.26 (Hypergeometric probability example)
   (d) Class example: Hung jury (See textbook: Example 3.2.5, another hypergeometric example)
   (e) HW problem 3.5.6 (Finding the expected value of a binomially distributed random variable)