Due on Thursday December 1 at the beginning of lecture.

Note: You may evaluate integrals or simplify sums using a symbolic calculator, software such as Maple or Mathematica (available in some UNR computer labs, or free online resources like Wolfram Alpha. Please note where you use such resources in your answers.

1. 3.12.20
2. 4.2.2
3. 4.2.17
4. 4.2.22 (461 only)
   4.2.25 (661 only)
5. 4.3.2(b, c)
6. 4.3.5(d)
7. 4.4.7
8. 4.5.6 (To clarify, “the number of trials in excess of $r$” is just the number of failures before the $r$th success.)
9. (Memoryless property exponential waiting times): Let $T$ be an exponential RV with rate $r$, i.e.,

$$f_T(t) = re^{-rt} \text{ and } F_T(t) = P(T \leq t) = 1 - e^{-rt}, \text{ for } r \geq 0$$

$T$ represents the waiting time until an event occurs. Suppose 10 minutes go by, and no event occurs. What is the distribution of the waiting time after the 10 minute mark? To answer this, first define $U$ to be the waiting time after 10 minutes, where $F_U(t) = P(U \leq t) = P(T \leq 10+t \mid T \geq 10)$. Use the CDF of $T$, and the definition of conditional probability, to show that $U$ is exponential with rate $r$. That is, show that

$$P(U \leq t) = 1 - e^{-rt}.$$ 

10. Recall Example 3.12.3 (pg 208, also done in class) which shows that the MGF for $T$, an exponential RV with rate $r$, is $M_T(t) = \frac{r}{r-t}$. See Theorem 4.6.5 (pg 273) which shows
that the MGF for $Y$, a gamma RV with shape parameter $n$ and rate $r$, is $M_Y(t) = \frac{r^n}{(r - t)^n}$.

Use the properties of moment generating functions spelled out in Theorems 3.12.2 and 3.12.3 (pg 214) to show that the sum of $n$ iid exponentials (with rate $r$) is a gamma distributed RV.

11. Let $X_1, \ldots, X_n$ be independent exponential RVs, with their own (different) rates $r_i$.

(a) Using the MGFs from #10 above, find $M_{X_i}(0)$, $M_{X_i}'(0)$ and $E[X_i]$.

(b) Let $Y = \sum_i X_i$. Using the properties of MGFs, and the generalized product rule for the product of $n$ differentiable functions $G_i(t)$,

$$\frac{d}{dt} \left( \prod_{i=1}^{n} G_i(t) \right) = \sum_{i=1}^{n} \left( G_i'(t) \prod_{j \neq i} G_j(t) \right)$$

find $M_Y'(0)$ to confirm the obvious relationship $E[Y] = \sum_i E[X_i]$

Reminder: For problems with answers given in the back of the text, show your work!