Problem 5.3.2

Let 52, 69, 73, 88, 87, 56 be the data describing female $CH_3^{203}$ half-life. Then in order to construct a 95\% C.I. for the true female $CH_3^{203}$ half-life, we need to compute $\bar{X}$. We assume that the female $CH_3^{203}$ half-life has a normal distribution with standard deviation $\sigma = 8$ days.

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{6} X_i = \frac{52 + 69 + 73 + 88 + 87 + 56}{6} = 70.8333$$

Thus, the 95\% C.I. is

$$(\bar{X} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{X} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}) = (70.8333 - 1.96 \frac{8}{\sqrt{6}}, 70.8333 + 1.96 \frac{8}{\sqrt{6}}) = (64.432, 77.234)$$

Note that 80 days (men’s average $CH_3^{203}$ half-life) does not lie within the 95\% C.I.. This is evidence that supports the claim that males and females metabolize methylmercury at different rates.

Problem 5.3.3

Recall that the length of the confidence interval is

$$L = 2Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}.$$ 

Thus,

$$L = 2 \times 1.96 \times 14.3 = \frac{56.056}{\sqrt{n}}.$$ 

In order to guarantee that $L < 3.06$, we must have

$$\frac{56.056}{\sqrt{n}} < 3.06 \quad \text{so} \quad n > \left(\frac{56.056}{3.06}\right)^2 = 335.58$$

Hence, $n = 336$.

Problem 5.3.4.

Suppose that the random variable $Y$ is normally distributed and that $\sigma$ is known.

(a) For $(\bar{y} - 1.64 \frac{\sigma}{\sqrt{n}}, \bar{y} + 2.33 \frac{\sigma}{\sqrt{n}})$ we have

$$P(\bar{y} - 1.64 \frac{\sigma}{\sqrt{n}} < \mu < \bar{y} + 2.33 \frac{\sigma}{\sqrt{n}}) = P(-1.64 \leq \frac{\bar{y} - \mu}{\sigma/\sqrt{n}} \leq 2.33) = P(-1.64 \leq Z \leq 2.33)$$

$$= P(Z \leq 2.33) - P(Z \leq -1.64) = 0.94.$$ 

Hence, a 94\% confidence level is associated with this interval.
(b) For \((-\infty, \bar{y} + 2.58 \frac{\sigma}{\sqrt{n}})\) we have

\[
P(-\infty < \mu < \bar{y} + 2.58 \frac{\sigma}{\sqrt{n}}) = P(-\infty \leq \frac{\bar{y} - \mu}{\sigma/\sqrt{n}} \leq 2.58) = P(-\infty \leq Z \leq 2.58) = 0.995.
\]

Hence, a 99.5\% confidence level is associated with this interval.

(c) For \((\bar{y} - 1.64 \frac{\sigma}{\sqrt{n}}, \bar{y})\) we have

\[
P(\bar{y} - 1.64 \frac{\sigma}{\sqrt{n}} < \mu < \bar{y}) = Pr(-1.64 \leq \frac{\bar{y} - \mu}{\sigma/\sqrt{n}} \leq 0) = Pr(-1.64 \leq Z \leq 0) = 0.45.
\]

Hence, a 45\% confidence level is associated with this interval.

**Problem 5.3.8.**

From Theorem 5.3.1, the confidence interval is

\[
\left(179\sqrt{\frac{220}{(179/220)(1-179/220)}} - 1.64 \frac{179}{220}, \left(179\sqrt{\frac{220}{(179/220)(1-179/220)}} + 1.64 \frac{179}{220}, \right)\right)
\]

\[= (0.771, 0.857)\]

**Problem 5.3.12.**

\[
\frac{x}{n} - 0.67 \sqrt{\frac{(x/n)(1-x/n)}{n}} = 0.57
\]

\[
\frac{x}{n} + 0.67 \sqrt{\frac{(x/n)(1-x/n)}{n}} = 0.63
\]

Adding the two equations gives \(2 \frac{x}{n} = 1.20\) or \(\frac{x}{n} = 0.60\)

Substituting the value for \(\frac{x}{n}\) into the first equation above gives

\[
0.60 - 0.67 \sqrt{\left(\frac{0.60(1-0.60)}{n}\right)} = 0.57.
\]

Solving this equation for \(n\) gives \(n = 120\).

**Problem 5.3.13.**

\[
2.58 \sqrt{\frac{p(1-p)}{n}} \leq 2.58 \sqrt{\frac{1}{4n}} = 0.01, \text{ so take } n \geq \frac{(2.58)^2}{4(0.01)^2} = 16.641
\]
Problem 5.3.25.

Take \( n \) to be the smallest integer \( \geq \frac{z_{10}^2}{4(0.02)^2} = \frac{1.28^2}{4(0.02)^2} = 1024. \)

Problem 5.3.26.

a) Take \( n \) to be the smallest integer \( \geq \frac{z_{0.025}^2}{4(0.03)^2} = \frac{1.44^2}{4(0.03)^2} = 576. \)

b) Take \( n \) to be the smallest integer \( \geq \frac{z_{0.025}^2 p(1-p)}{(0.03)^2} = \frac{1.44^2(0.10)(0.90)}{(0.03)^2} = 207.36 \), so let \( n = 208. \)