Problem 7.3.4

Let \( Y = \frac{(n-1)S^2}{\sigma^2} \). Then \( \text{Var}(Y) = \frac{(n-1)^2 \text{Var}(S^2)}{\sigma^4} \). We also know that \( \text{Var}(Y) = \text{Var}(\chi^2_{n-1}) = 2(n-1) \). It follows that \( \text{Var}(S^2) = \frac{2\sigma^4}{n-1} \).

Problem 7.3.7

(a) 0.983  (b) 0.132  (c) 9.00

Problem 7.3.12.

If \( P(a \leq F_{m,n} \leq b) = q \), then \( P \left( \frac{1}{b} \leq F_{m,n} \leq \frac{1}{a} \right) = P \left( \frac{1}{b} \leq F_{n,m} \leq \frac{1}{a} \right) = q \). Thus, \( c = \frac{1}{b} \) and \( d = \frac{1}{a} \).

Checking this property with the example given:

\[
P(0.052 \leq F_{2,8} \leq 4.46) = P \left( \frac{1}{4.46} \leq F_{8,2} \leq \frac{1}{0.052} \right) = P(0.224 \leq F_{8,2} \leq 19.4) = 0.9.
\]

Problem 7.4.8.

Given that \( n = 7 \) and \( \alpha = 0.05 \), we find that \( t_{0.025,6} = 2.4469 \). Here \( \sum_{i=1}^{n} y_i = 12,808 \) and \( \sum_{i=1}^{n} y_i^2 = 25,540,436 \) so \( \bar{y} = \frac{12,808}{7} = 1829.71 \) and \( s = \sqrt{\frac{7(25,540,436) - (12,808)^2}{7(6)}} = 719.43 \). The confidence interval is

\[
\left( 1829.71 - 2.4469 \frac{719.43}{\sqrt{7}}, 1829.71 + 2.4469 \frac{719.43}{\sqrt{7}} \right) = (1164.35, 2495.07)
\]

Problem 7.5.2.

(a) 0.95  (b) 0.90  (c) 0.975 - 0.025 = 0.95  (d) 0.99

Problem 7.5.8.

If \( n = 19 \) and \( \sigma^2 = 12.0 \), then \( \frac{18 \cdot S^2}{12.0} \) has a \( \chi^2 \) distribution with \( n - 1 = 18 \) degrees of freedom, so

\[
P \left( 8.231 \leq \frac{18 \cdot S^2}{12.0} \leq 31.526 \right) = 0.95 = P(5.49 \leq S^2 \leq 21.02)
\]
Problem 7.5.16.

(a) Test $H_0 : \mu = 10.1$ versus $H_1 : \mu > 10.1$. The test statistic is

$$\frac{\bar{y} - \mu_0}{s/\sqrt{n}} = \frac{11.5 - 10.1}{10.17/\sqrt{24}} = 0.674.$$ 

The critical value is $t_{\alpha, n-1} = t_{0.05, 23} = 1.7139$. Since $0.674 < 1.7139$, we accept the null hypothesis. Thus, we cannot ascribe the increase of the portfolio yield over the bench mark to the analyst’s system for choosing stocks.

(b) Test $H_0 : \sigma^2 = 15.67$ versus $H_1 : \sigma^2 < 15.67$. The test statistic is

$$\chi^2 = \frac{23(10.17)^2}{15.67^2} = 9.688.$$ 

The critical value is $\chi^2_{0.05, 23} = 13.091$. Since $9.688 < 13.091$, we reject the null hypothesis. The analyst’s method of choosing stocks does seem to result in less volatility.