Detection Theory

Detection Problem: Decide based on noisy measurements with a measure of confidence
1. If an event occurred or not.
2. Which of a number of possible outcomes has occurred?
Statistics: called decision theory.

Outline
- What is detection theory?
- Applications of detection theory.
- Classes of detection problems.
- Examples.
- Estimation theory.

Example Applications of Detection Theory
- Radar/sonar.
- Communications.
- Speech/image processing.
- Biomedicine.
- Fault detection.
- Seismology.

Radar/Sonar
- Detect the presence of a target.
- Transmit and electromagnetic pulse.
- Test received noisy signal:
  * If a pulse is detected, it was reflected from the target and a target is present.
  * If no pulse is detected then no target is present.

Digital Communication
- Binary Phase Shift Keying (BPSK)
- Digital data source gives a “0” or “1”.
- Modulate signal, transmit it, receive it, then demodulate it.
- Determine the transmitted signal (“0” or “1”) from the demodulated signal.
Classes of Detection Problems
Detect signals from noisy measurements
1. Known signals in additive noise.
2. Deterministic signals with unknown parameters in additive noise.
3. Random signals in additive noise.
   • We assume additive Gaussian noise.

Known Signals
- Using one or more noisy measurements decide
  1. If the known signal “s” is present.
  2. Which of the known signals \( s_i, i = 1, \ldots, N \), is present?

Deterministic Signals with Unknown Parameters
- Using one or more noisy measurements decide
  1. If the signal “s” is present.
  2. Which of the signals \( s_i, i = 1, \ldots, N \), is present?
- One or more signal parameters unknown (amplitude, frequency, etc.)

Random Signals
- Using one or more noisy measurements decide
  1. If the signal “s” is present.
  2. Which of the signals \( s_i, i = 1, \ldots, N \), is present?
- One or more random signal parameters unknown (amplitude, frequency, etc.) but statistical information about it known.

Estimation
Point Estimation: estimate the value of a scalar of vector.
Interval Estimation: estimate the interval in which a scalar or vector lies with a specified confidence level
\[ P(x_1 < X < x_2) = \gamma \]

Applications to Radar/Sonar
- Frequency estimation: estimate Doppler shift.
- Amplitude estimation: estimate target strength, size, distance.
- Phase/BW Estimation: target identification.
- Spectral estimation: target identification.
- Signal estimation: optimal filter to estimate signal from noisy measurements (Kalman filter).