Combinatorics

M. Sami Fadali
Professor of Electrical Engineering
University of Nevada, Reno

Outline

• Combinatorics
• Permutations
• Combinations

Combinatorics

Study of the arrangement of objects of a set into patterns.

1. Existence of arrangements (conditions).
2. Enumeration or classification of arrangements (counting and classifying).
3. Study of the properties of a known arrangement.
   e.g. magic squares: for what $n$ does it exist?

Permutations

$r$-permutation of an $n$-element set:

Ordered arrangements of $r$ of $n$ elements.

\[
P(n, r) = \begin{cases} 
\frac{n!}{(n-r)!} = n(n-1)\cdots(n-r+1), & n \geq r \\
0, & n < r 
\end{cases}
\]

\[
P(n, n) = n!
\]

Proof: $n$ ways to choose 1st element, $n - 1$ for 2nd, $n - r + 1$ for $r$th. Product = $P(n, r)$
Example: 5-Permutations, 52 Cards

\[ X = \{2,3,\cdots,10,J,Q,K,A\} \]
\[ \{10,J,Q,K,A\} \]
\[ \{A,10,J,Q,K\} \]
\[ \vdots \]
\[ \frac{52!}{(52-5)!} = 52 \times 51 \times 50 \times 49 \times 48 \]
\[ = 311,375,200 \quad 5-\text{permutations} \]

Combinations

\( r \)-combination of an \( n \)-element set:

Unordered selection of \( r \) of \( n \) elements

\[
C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} \quad n \geq r, \quad n < r
\]

\[ C(n,n) = C(n,0) = 1 \quad C(n,1) = n \]

Proof: \( P(n,r) = r! \) ways to arrange \( r \) elements,
\[ P(n,r) = r! \quad C(n,r) = \frac{n!}{(n-r)!} \]

Example: Permutations of 5 Cards

\[ X = \{10,J,Q,K,A\} \]
\[ \{10,J,Q,K,A\} \]
\[ \{A,10,J,Q,K\} \]
\[ \vdots \]
\[ 5! \quad \text{permutations} \]

Example: 5-Card Hands

\[ X = \{2,3,\cdots,10,J,Q,K,A\} \]
\[ \{10,J,Q,K,A\} \]
\[ \{A,10,J,Q,K\} \]
\[ \vdots \]
\[ \binom{52}{5} = \frac{52!}{5!(48)!} \]
\[ = \frac{(52)(51)(50)(49)(48)}{(5)(4)(3)(2)(1)} \]
\[ = 2,598,960 \quad \text{combinations} \]
Conclusion

• Need to count number of subsets for probability determination.
• Use combinations and permutations for counting subsets.

Ref.