Complementary Filter

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Limitation of Wiener Filter

• Signal and noise must both be random.
• Many applications have a deterministic signal and random noise.
• Extend Wiener filter (or Phillips approach) to allow deterministic signal.

Example: Aircraft Position

• \( s(t) = \) position of aircraft in flight (scalar)
• Use sensor to determine the position.
• Position is deterministic.
• Measurement includes random errors.
• Need second sensor to use Wiener filter.

Instrumentation Application

• Use two measurements of the same signal.
• Use a different filter for each signal.
• Add the two filtered signals.
• Select filter transfer functions to minimize mean square error.
Selection of Two Filters

Attempt to make overall output approximately equal to the signal.

\[ X(s) = G_1(s)[S(s) + N_1(s)] + G_2(s)[S(s) + N_2(s)] \]
\[ = [G_1(s) + G_2(s)]S(s) + G_1(s)N_1(s) + G_2(s)N_2(s) \]
\[ = S(s) + [1 - G(s)]N_1(s) + G(s)N_2(s) \]
\[ = [S(s) + N_1(s)] + G(s)[N_2(s) - N_1(s)] \]

Block Diagrams

Complementary Filter Properties

- Signal unaffected by choice of filter \( G(s) \)
- Noise affected by choice of filter.
- If \( N_1(s) \) low frequency and \( N_2(s) \) high frequency, use a low pass filter \( G(s) \) then \([1 - G(s)]\) is a high pass filter.
- Input to \( G(s) \) is a purely random signal
  - Estimate \( N_1(s) \) using a Wiener filter
  - \( N_2(s) \) noise

Example: Position Servo

- Tachometer provides noisy velocity measurement.
- Noisy accelerometer measurement.
- Assume: need a LPF for the tachometer signal.

\[ G(s) = \frac{1}{Ts + 1} \quad 1 - G(s) = \frac{Ts}{Ts + 1} \]
Example (Cont.)

- Minimize error to obtain the optimum filter.
- Optimum linear filter of selected form.
- Use (causal) Wiener filter for the optimum linear filter.