Sampling

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Outline

- Sampling.
- Band limiting.
- Time limiting.
- Discrete Fourier Transform.
- Fast Fourier Transform.

Sampling

- Sample continuous time signals for signal processing.
- Nyquist frequency: twice the highest (significant) frequency of the signal.
- Physical signals are not bandlimited.

Band-limited Function

\[
F\{g(t)\} = G(\omega) = \begin{cases} 
\text{nonzero}, & |\omega| \leq 2\pi W \\
0, & |\omega| > 2\pi W 
\end{cases}
\]

- Idealization
- Approximately true in practice.
Sampling

- Sample at the Nyquist rate or faster.
- Multiplication with impulse train.

\[ g^*(t) = \sum_{n=-\infty}^{\infty} g(nT)\delta(t - nT), \quad T \leq \frac{1}{2W} \]

Ideal Low Pass Filter

- Idealized filter.
- Bandwidth \( W \).

\[ h(t) = \frac{\sin(2\pi W t)}{2\pi W t} \]

\[ H(j\omega) = \begin{cases} \frac{1}{2W}, & |\omega| \leq 2\pi W \\ 0, & |\omega| > 2\pi W \end{cases} \]

LPF Sampled Signal

- Convolution of sampled signal, \( T = 1/(2W) \), with sinc (ideal LPF).

\[ g(t) = h(t) \ast g^*(t) \]

\[ = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\frac{\sin(2\pi W t - n\pi)}{2\pi W t - n\pi}, \quad T \leq \frac{1}{2W} \]

- Can (in theory) recover signal from its samples.
- Shannon’s Theorem

Time-limited Signal

- Time-limiting: multiplication with pulse in the time domain.
- Frequency convolution with sinc: not bandlimited.

\[ G(\omega) = \sum_{n=-\infty}^{\infty} G \left(\frac{2\pi n}{T}\right) \frac{\sin(\omega T/2 - n\pi)}{\omega T/2 - n\pi} \]
Discrete Fourier Transform

- Discrete time $g_i, i = 0, 1, ..., N - 1$
- Discrete frequency $G_i, i = 0, 1, ..., N - 1$
- Periodic waveform: repeat record.

$$G_n = \frac{1}{N} \sum_{k=0}^{N-1} g_k \exp \left[ j \frac{2\pi k}{N} \right], n = 0, 1, ..., N - 1$$

- Like Fourier series.

Fast Fourier Transform

- FFT Algorithms: Cooley & Tukey
- Efficient computation of discrete Fourier transform.
- Can be generalized to complex sequences.