Consider the closed-loop dc servo system

\[ P(s) = \frac{K_c}{s^2 + (R/L)s + (K_c/J)} \]

with proportional control gain \( k \) and uncertain parameters \( q = \{K_c, L, R\} \)

\( K_c \in [0.04, 0.06], \quad L = 0.05, \quad N = 12 \)

Determine the stable range of controller gain

a) Using overbounding and the simplified Kharitonov test.

b) Using the Routh-Hurwitz criterion directly.

Solution

The closed-loop characteristic polynomial for the system is

\[ s^4 + \frac{2kK_c}{J} s^3 + \frac{K_c}{J} s^2 + \frac{K_c}{J} + k \frac{K_c}{J} \]

If the coefficients are allowed to vary independently, we obtain the coefficients

\[ a_4 = R/L, \quad a_3 = \frac{N2K_c}{J}, \quad a_2 = \frac{K_c}{J}, \quad a_1 = k \frac{K_c}{J} \]

\[ a_0 = \frac{K_c}{J} \]

Overbounding allows us to use tests of interval polynomials in more complex cases. However, if the test indicates instability, the family may or may not be unstable since the testing set is larger than the family.

Examples

Example 1 (Belanger, p. 214) Assess the robust stability of the third-order interval family with coefficients

\[ a_0 = [1, 2], a_1 = [0.5, 1], a_2 = [2, 3], a_3 = 1. \]

Solution

For a third order polynomial, only \( D^* \) is needed.

\[ D^*(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0 \]

For \( a_3 = 2 \) and \( a_2 = 0.5 \), we reach the same conclusion by obtaining the roots of the polynomial.

\[ \text{roots}(dpm) \]

Using MATLAB, we reach the same conclusion by obtaining the roots of the polynomial.

\[ \text{roots}(dpp) \]

There are two sign changes in the first column which indicates an unstable polynomial with two RHP poles. Hence the family is not robustly stable.

Example 2 (Belanger, p. 215) Overbounding

Consider the closed-loop dc servo system

\[ P(s) = \frac{K_c}{s^2 + (R/L)s + (K_c/J)} \]

Using the Routh-Hurwitz criterion directly.

\[ D(s, q) = s^4 + \frac{2kK_c}{J} s^3 + \frac{K_c}{J} s^2 + \frac{K_c}{J} + k \frac{K_c}{J} \]

If the coefficients are allowed to vary independently, we obtain the coefficients

\[ a_4 = R/L, \quad a_3 = \frac{N2K_c}{J}, \quad a_2 = \frac{K_c}{J}, \quad a_1 = k \frac{K_c}{J} \]

\[ a_0 = \frac{K_c}{J} \]

The first column is positive and the system is robustly stable for \( k < 3.815 \).