Lag Design

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Outline

- Why use lag compensation?
- Lag transfer function.
- Design procedure.
- Example.

Why use lag compensation?

- Attempt to meet specifications using proportional control.
- Transient response specifications met.
- Steady-state specifications not met.
- Add lag to improve steady-state error.

Lag Transfer Function

\[ G_c(s) = \beta \frac{T_s + 1}{\beta T_s + 1} = \frac{s + 1/T}{s + 1/(\beta T)} \]

\( \beta > 1 \)

\( \beta \) = Improvement in error constant
**Design Procedure**

1. Design a proportional control for the system to meet the transient response specs.
2. Calculate the necessary improvement in the steady-state error.
3. Calculate the necessary gain increase $\beta$.
4. Add a zero at $-\zeta \omega_n / 10$ pole is at $-\zeta \omega_n / (10\beta)$

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**Example**

- Process transfer function

$$G(s) = \frac{1}{(s+2)(s+4)(s+10)}$$

- Design closed-loop unity feedback system to obtain 10% overshoot and 10% error.

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**Proportional Control Design**

```matlab
>> zeta=abs(log(.1))/sqrt(abs(log(.1))^2+pi^2)
0.5912
```

- Use $\zeta = 0.6$
- Obtain the closed-loop pole and the gain from the MATLAB root locus plot.

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**SISOTOOL Plots**

$$s_{1,2} = -2.33 \pm j3.1$$
**Step Response: Proportional Control K=90.1**

![Step Response Graph]

- **System:** Closed Loop r to y
- **I/O:** r to y
- **Settling Time (sec):** 1.63
- **Time (sec):** 2.48
- **Amplitude:** 0.53
- **Peak amplitude:** 0.578
- **Overshoot (%):** 8.72
- **At time (sec):** 1.13

**Steady-state Error**

- **Type 0**
- **Position error constant**
  \[ K_p = \lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{90.1}{(s+2)(s+4)(s+10)} = 1.126 \]
  \[ e(\infty)\% = \frac{100}{1+K_p} = \frac{100}{1+1.126} = 47\% \]

**Compensator**

- Let \( \beta = 8 \)
- Zero at \(-\zeta \omega_n/10 \approx -0.2, T = 5 s\)
- \( \beta T = 40 s \)
- In addition to amplifier gain needed for proportional control design
  \[ G_c(s) = 90.1 \times 8 \frac{5s + 1}{40s + 1} = 90.1 \frac{s + 0.2}{s + 1/40} \]
**Comments**

- Bad response and huge gain.
- Lag-lead needed to reduce the error while keeping the transient response acceptable.