Empirical Tuning of PID Controllers

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Outline

• Why empirical tuning?
• Procedure for empirical tuning.
• Example

Block Diagram

Empirical Tuning

• Allows us to address
  (i) load disturbance rejection specifications
  (ii) time delay in the process.
• PID controller transfer function:

\[ C(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) \]

\[ T_i = \frac{K_p}{K_i}, \quad T_d = \frac{K_d}{K_p} \]

\( T_i \) = integral time constant
\( T_d \) = derivative time constant.
**Plant Model**

Use a simple first order model with time delay

\[ G(s) = \frac{K}{\tau s + 1} e^{-Ls} \]

- \( K \) = process gain
- \( \tau \) = process (dominant) time constant
- \( L \) = (apparent) dead time of the process.

**Tangent Method**

1. Obtain the step response experimentally.
2. Draw a tangent at the inflection point.
3. Gain \( K = \frac{\text{steady-state change in the output}}{\text{amplitude of input step}} \)
4. Dead time \( L \) = from time of step input to the intersection of the tangent line with the time axis.
5. \( \tau + L \) = time interval between the step input and the intersection of the tangent line with the final steady-state output level.

**Alternatively**

5. \( \tau + L \) = time interval between the step input and the time when the process output attains the 63.2% of its final value.

Note: If the dynamics of the process can be perfectly described by a first-order-plus-dead-time model, then the values of \( \tau \) obtained in the two cases are identical.
Ziegler-Nichols Tuning Rules

<table>
<thead>
<tr>
<th>Controller type</th>
<th>$K_p$</th>
<th>$T_i$</th>
<th>$T_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$\frac{\tau}{KL}$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>PI</td>
<td>$\frac{0.9\tau}{KL}$</td>
<td>$3L$</td>
<td>$-$</td>
</tr>
<tr>
<td>PID</td>
<td>$\frac{1.2\tau}{KL}$</td>
<td>$2L$</td>
<td>$0.5L$</td>
</tr>
</tbody>
</table>

Example

Process transfer function $G(s) = \frac{1}{(s+1)^4} e^{-0.2s}$

>> g=zpk([],[-1,-1,-1,-1],1,'inputdelay',0.2)

zero/pole/gain:

$1$

$\exp(-0.2s) * \frac{1}{(s+1)^4}$

Estimate a first-order-plus-dead-time model of the process and design a PID controller by applying the Ziegler-Nichols tuning rules.

Block Diagram

Apply tangent method

$KA = 1$

$A = 1$

$K = 1$

$L = 1.55, \tau + L = 4.55 \Rightarrow \tau = 3$
PID Controller

- Use table to calculate the controller parameters

\[ K_p = \frac{1.2\tau}{KL} = 2.32 \]
\[ T_i = 2L = 3.1 \]
\[ T_d = 0.5L = 0.775 \]

\[ C(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) \]
\[ = 2.32 \left( 1 + \frac{1}{3.1s} + 0.775s \right) \]

Process Output PID Control

- Oscillatory, typical feature of the Ziegler-Nichols method.
- Minimize effect of the disturbance on the step response.