1. Prove that $\sqrt{3}$ is an irrational number.

   **Proof.** Suppose $\sqrt{3}$ is equal to the rational number $p/q$, with $p, q$ not sharing common divisor bigger than 1. Then $3q^2 = p^2$, which means $p$ is divisible by 3. Let $p = 3p_1$. Then we have $3p_1^2 = q^2$. Now $q$ must be divisible by 3 as well. But this contradicts with the assumption that $p, q$ do not share common divisor bigger than 1. So $\sqrt{3}$ is irrational.

2. Show that in a party of 37 people, there are at least 4 people whose birthdays are in the same month.

   **Proof.** Suppose otherwise, for each month there are at most three people whose birthdays are in that month. Then the total number of people is at most $3 \times 12 = 36$, which contradicts with the fact that there are 37 people.