1. Find all solutions of each of the following system of congruences.

(a) \[
\begin{aligned}
&x \equiv 1 \pmod{3} \\
&x \equiv 3 \pmod{5} \\
&x \equiv 5 \pmod{7}
\end{aligned}
\]

Solution. 
\[
\begin{aligned}
&c_1 = 1, c_2 = 3, c_3 = 5 \\
n_1 = 35, n_2 = 21, n_3 = 15 \\
\bar{n}_1 = 2, \bar{n}_2 = 1, \bar{n}_3 = 1 \\
3 \times 5 \times 7 = 105
\end{aligned}
\]

So by the Chinese Remainder Theorem, we get the solution
\[
x \equiv 1 \times 35 \times 2 + 3 \times 21 \times 1 + 5 \times 15 \times 1 \equiv 103 \pmod{105}.
\]
Alternatively, one may as well write the solution as \(x \equiv -2 \pmod{105}\).

(b) \[
\begin{aligned}
&4x \equiv 2 \pmod{6} \\
&3x \equiv 5 \pmod{7} \\
&2x \equiv 4 \pmod{11}
\end{aligned}
\]

Solution. The first equation can be restated as \(2x \equiv 1 \pmod{3}\).
\[
\begin{aligned}
&c_1 = 2, c_2 = 4, c_3 = 2 \\
n_1 = 77, n_2 = 33, n_3 = 21 \\
\bar{n}_1 = 2, \bar{n}_2 = 3, \bar{n}_3 = -1 \\
3 \times 7 \times 11 = 231
\end{aligned}
\]

So by the Chinese Remainder Theorem, we get the solution
\[
x \equiv 2 \times 77 \times 2 + 4 \times 33 \times 3 + 2 \times 21 \times (-1) \equiv 200 \pmod{231}.
\]
Alternatively, one may as well write the solution as \(x \equiv -31 \pmod{231}\).