Record multiple choice answers below

(1) (A) (B) (C) (D) (E) (F)
(2) (A) (B) (C) (D) (E) (F)
(3) (A) (B) (C) (D) (E) (F)
(4) (A) (B) (C) (D) (E) (F)
(5) (A) (B) (C) (D) (E) (F)
(6) (A) (B) (C) (D) (E) (F)
(7) (A) (B) (C) (D) (E) (F)
(8) (A) (B) (C) (D) (E) (F)
(9) (A) (B) (C) (D) (E) (F)
(10) (A) (B) (C) (D) (E) (F)
(11) (A) (B) (C) (D) (E) (F)
(12) (A) (B) (C) (D) (E) (F)
(13) (A) (B) (C) (D) (E) (F)
(14) (A) (B) (C) (D) (E) (F)
Multiple Choice Section

Each question in this section is worth 5 points. You can write on these pages. Mark the answers on the right for your own use. Record the answers on page 1 for grading.

(1) 17 does not divide ______
(A) 34  (B) -17  (C) 51  (D) 68  (E) 7  (F) none of these.

(2) 105 \equiv ______ \pmod{13}
(A) 6  (B) 1  (C) 7  (D) 5  (E) 9  (F) none of these.

\[ 105 = 13 \times 8 + 1 \]

(3) The number of primes between 20 and 40 is ______
(A) 8  (B) 9  (C) 10  (D) 11  (E) 12  (F) none of these.

\[ 23, 29, 31, 37 \]

(4) The greatest common divisor of 63 and 42 is ______
(A) 6  (B) 9  (C) 7  (D) 21  (E) 1  (F) none of these.
(5) The number of integer solutions to the linear Diophantine equation $15x - 45y = 5$ is ______
(A) 0    (B) 1    (C) 2    (D) 3     (E) infinite     (F) none of these.

\[ \gcd(15, 45) = 15 \]

(6) If $a \equiv 2 \pmod{6}$ and $b \equiv 3 \pmod{6}$, then $12a + 4b \equiv ____ \pmod{6}$.
(A) 0    (B) 1    (C) 2    (D) 3    (E) 4    (F) 5.

\[ 12a + 4b \equiv 4b \equiv 4 \times 3 \equiv 12 \equiv 0 \pmod{6} \]

(7) The inverse of 7 modulo 11 is ______
(A) 1    (B) 3    (C) 8    (D) 9    (E) 10    (F) none of these.

\[ 7 \times 8 \equiv 56 \equiv 1 \pmod{11} \]

(8) The number $17! + 1$ is divisible by ______
(A) 18    (B) 14    (C) 15    (D) 16    (E) 17    (F) none of these.

18 is not a prime
So by Wilson's Thm, $17! + 1 \not\equiv 0 \pmod{18}$

$17! + 1$ is not divisible by 14, 15, 16, 17 either.
(9) If \( a \equiv -2 \pmod{5} \), then \( a^{2016} \equiv ____ \pmod{5} \)
(A) 0  (B) 1  (C) 2  (D) 3  (E) 4  (F) none of these.

\[ \gcd(-2, 5) = 1 \]

Fermat's Thm \( \Rightarrow (-2)^4 \equiv 1 \pmod{5} \)

\[ 5 \frac{5^{2016}}{2} \equiv (-2)^{2016} \equiv \left[ (-2)^4 \right]^{504} \equiv 1 \pmod{5} \]

(10) If \( m \) is an integer, then a complete residue system modulo \( m \) must consists of ____ numbers.
(A) 0  (B) 1  (C) \( m - 1 \)  (D) \( m \)  (E) \( m^2 \)  (F) none of these.

(11) Which of the following is NOT a complete residue system modulo 5?
(A) \{0, 1, 2, 3, 4\}  (B) \{6, 7, 8, 9, 10\}  (C) \{0, 5, 10, 15, 20\}
(D) \{3, 6, 9, 12, 15\}  (E) \{-2, -1, 0, 1, 2\}  (F) none of these

\[ 0 \equiv 5 \equiv 10 \equiv 15 \equiv 20 \pmod{5} \]
(12) Which of the following is NOT a reduced residue system modulo 5?

(A) \{1, 2, 3, 4\}  
(B) \{11, 12, 13, 14\}  
(C) \{2, 4, 8, 16\}  
(D) \{4, 8, 12, 16\}  
(E) \{3, 9, 27, 81\}  
(F) \{1, 9, 17, 36\}.

\[ 1 \equiv 6 \pmod{5} \]

(13) Recall that \( \phi(m) \) is Euler's phi function. Then \( \phi(22) = \)_____

(A) 10  
(B) 9  
(C) 8  
(D) 7  
(E) 6  
(F) none of these.

(14) Which of the following statement is FALSE?

(A) If \( p \) divides \( (p - 1)! + 1 \), then by Wilson's theorem \( p \) must be a prime.

(B) If \( ac \equiv bc \pmod{m} \) and \( \gcd(c, m) = 1 \), then \( a \equiv b \pmod{m} \).

(C) If \( a \equiv b \pmod{m} \) and \( n \) divides \( m \), then \( a \equiv b \pmod{n} \).

(D) If the integers \( a \) and \( m \) are coprime, then there exists an integer \( \bar{a} \) such that \( a \cdot \bar{a} \equiv 1 \pmod{m} \).

(E) The linear diophantine equation \( 16x + 24y = 256 \) has only one integer solution.

(F) If \( p \) is a prime and \( p | ab \), then either \( p | a \) or \( p | b \).
1 (10 pts) Use Euclid’s division algorithm to find the general solution of the linear Diophantine equation

\[ 4x + 11y = 5. \]

\[
\begin{align*}
1 & = 4 \times 2 + 3 \\
4 & = 3 \times 1 + 1 \\
1 & = 4 - 3 = 4 - (11 - 4x) = 4x + 11x(-1) \\
\text{So} \quad 5 & = 4 \times 15 + 11x(-5) \\
x & = 15 \\
y & = -5
\end{align*}
\]

The general sol should be \( \begin{align*}
x & = 15 + 11t \\
y & = -5 - 4t
\end{align*} \) + is an integer.

2 (10 pts) Find the last digit of \( 33^{54} \).

Need to find the residue of \( 33^{54} \) modulo 10.

Recall that \( \varphi(10) = 4 \). So by Euler’s Thm,

\[
33^4 \equiv 1 \pmod{10}
\]

Hence \( 33^{54} \equiv 33^{4 \times 13 + 2} \equiv (33^4)^{13} \times 33^2 \equiv 1 \times 3^2 \equiv 9 \pmod{10} \).

Last digit of \( 33^{54} \) is 9.

3 (10 pts) Solve the linear congruence equation

\[ 5x \equiv 2 \pmod{13}. \]

Notice that \( 5 \times 8 = 40 \equiv 1 \pmod{13} \).

So multiply by 8 on both sides, and get

\[ 8 \times 5x \equiv 8 \times 2 \pmod{13} \]

\[ x \equiv 3 \pmod{13} \]