82.4: 1.
(a) \(357 = 3 \times 7 \times 17\) \hspace{0.5cm} 629 = 17 \times 37
\[\Rightarrow \left[357, 629\right] = 3 \times 7 \times 17 \times 37 = 13209\]

(b) \([-357, 629] = 13209\)

Note: \(GCD\) and \(LCM\) are always positive by definition.

(c) \(299 = 13 \times 23\) \hspace{0.5cm} 377 = 13 \times 29
\[\Rightarrow \left[299, 377\right] = 13 \times 23 \times 29 = 8671\]

6. Proof. Let \(m = \left[a, b\right]\). Then \(a/m\) and \(b/m\).

So \(ac|m\) and \(bc|m\), i.e. \(cm\) is a common divisor of \(ac\) and \(bc\).

If \(ac\mid n\) and \(bc\mid n\), then \(a/\frac{n}{c}\) and \(b/\frac{n}{c}\) which means \(\frac{n}{c}\) is a common multiple of \(a\) and \(b\). By Thm 2.18, we have \(m \mid \frac{n}{c}\), i.e. \(cm \mid n\).

So by Thm 2.18 again, \([ac, bc] = cm = c \left[a, b\right]\).

Note: Alternatively, one may use Ex 9 in §2.3 and Thm 2.19.
8. \textbf{Proof.} Since \(2(9n+8) - 3(6n+5) = 1\)

we know \((9n+8, 6n+5) = 1\) by Thm 2.6.

Now by Thm 2.19, we have

\[
\left\lceil 9n+8, 6n+5 \right\rceil = \frac{(9n+8) \cdot (6n+5)}{(9n+8, 6n+5)} = 54n^2 + 93n + 40.
\]

82.5 = 1.

(a) 4725 = \(3^3 \times 5^2 \times 7\)

(b) 3718 = \(2 \times 11 \times 13^2\)

(c) 3234 = \(2 \times 3 \times 7^2 \times 11\)

3. \((3718, 3234) = 2 \times 11 = 22\)

\[
\left\lceil 3718, 3234 \right\rceil = 2 \times 3 \times 7^2 \times 11 \times 13^2 = 546546
\]

5. \(\tau(3718) = (1+1) \cdot (1+1) \cdot (2+1) = 12\)

\(
\sigma(3718) = (1+2) \cdot (1+1) \cdot (1+13+13^2) = 6588
\)

10. Recall Thm 2.25, \((a, b) = \prod_{i=1}^{r} p_i^{u_i}\) where \(u_i = \min(a_i, b_i)\).

So \((a, b) = 1\) if and only if \(u_i = 0\) for all \(i\). But this is so if and only if one of \(a_i\) and \(b_i\) is 0 for all \(i\), i.e.

\(a_i b_i = 0\) for all \(i\).