Multiple Choice Section

(1) The number of integer solutions to the linear Diophantine equation $12x - 21y = 4$ is ______
   
   (A) 0  
   (B) 1  
   (C) 2  
   (D) 3  
   (E) infinite  
   (F) none of these.

(2) If $a \equiv 4 \pmod{5}$ and $b \equiv 3 \pmod{5}$, then $2a + 3b \equiv$ ______ \pmod{5}.

   (A) 0  
   (B) 1  
   (C) 2  
   (D) 3  
   (E) 4  
   (F) 5.

(3) The inverse of 7 modulo 11 is ______

   (A) 1  
   (B) 3  
   (C) 8  
   (D) 9  
   (E) 10  
   (F) none of these.
(4) If \( m \) is an integer, then a complete residue system modulo \( m \) must consists of ______ numbers.

(A) 0  (B) 1  (C) \( m - 1 \)  (D) \( m \)  (E) \( m^2 \)  (F) none of these.

(5) Which of the following is NOT a complete residue system modulo 5?

(A) \{0, 1, 2, 3, 4\}  (B) \{6, 7, 8, 9, 10\}  (C) \{0, 5, 10, 15, 20\}
(D) \{3, 6, 9, 12, 15\}  (E) \{-2, -1, 0, 1, 2\}  (F) none of these

(6) Which of the following is NOT a reduced residue system modulo 5?

(A) \{1, 2, 3, 4\}  (B) \{11, 12, 13, 14\}  (C) \{2, 4, 8, 16\}
(D) \{4, 8, 12, 16\}  (E) \{3, 9, 27, 81\}  (F) \{1, 9, 17, 36\}.
1 Use the Chinese Remainder Theorem to solve the linear diophantine system
\[
\begin{align*}
3x &\equiv 2 \pmod{5} \\
4x &\equiv 5 \pmod{7} \\
5x &\equiv 7 \pmod{11}
\end{align*}
\]
2 Find the prime factorization for 31!. 
3 Let $p = 5$ and $q = 11$. So $n = pq = 55$ and $\phi(n) = (p - 1)(q - 1) = 40$. Bob chooses his public exponent $e = 27$ which is coprime with 40. Then he publishes his public key $(55, 27)$ openly.

(a) Compute Bob’s private key.

(b) Suppose that Alice wants to send an integer 15 to Bob. Compute the cyphertext $c$ she will send in the open channel.

(c) After receiving the cyphertext $c$, check that Bob is able to recover the original integer 15 using his private key.
4 Use *Euclid's division algorithm* to find the general solution of the linear Diophantine equation

\[ 6x + 11y = 3. \]
5 Solve the linear congruence equation

\[ 7x \equiv 5 \pmod{15}. \]