"Show work" for full credit problems

1. Compute \((1 - i\sqrt{3})^6\).

2. Use the formula \(e^{i\theta} = \cos \theta + i \sin \theta\) to prove the double angle formula \(\cos 2\theta = \cos^2 \theta - \sin^2 \theta\) and \(\sin 2\theta = 2 \sin \theta \cos \theta\).

3. Show that \(f(z) = \sum_{n=1}^{\infty} (n + 1)^2 z^n\) is continuous in the open unit disk \(|z| < 1\). Is \(f(z)\) continuous at any point on the unit circle \(|z| = 1\)? Explain why.
4 Find the solutions to the equation $z^6 = -1$ in polar form.

5 Express $\frac{(2-3i)(1+2i)}{3+4i}$ in the form $a + bi$.

6 Suppose that $f(z) = \sum_{n=0}^{\infty} C_n z^n$ is a power series such that $f(1/n) = 5 - 1/n^2$ for all natural number $n$. Find $C_0, C_1$ and $C_2$. 

7 Solve the equation \( z^2 + \sqrt{32}iz - 6i = 0 \).

8 (a) Show that \( f(z) = x^3 + iy^3 \) is differentiable at all points on the line \( y = x \) and \( y = -x \).  
(b) Show that it is nowhere analytic.

9 Find all solutions of \( e^z = i \).