1. For any $t > 0$ denote $A_t = (-t, 2 + t)$.
   a. Determine $A_3 \setminus A_1$.
   b. Determine $\bigcap_{n=1}^{10} A_{1/n}$
   c. Prove that $\bigcap_{t>0} A_t = [0, 2]$.

2. Let $f : A \to B$ and $g : B \to C$ be functions. Prove
   a. If $g \circ f$ is onto, then $g$ is onto.
   b. If $g \circ f$ is one-to-one, then $f$ is one-to-one.

3. Consider the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ and the sets $X = (-1, 4)$, $Y = [1, 4]$.
   a. Determine $f^{-1}(X)$ and $f^{-1}(Y)$.
   b. Determine $f(f^{-1}(X))$ and $f(f^{-1}(Y))$.

4. Prove that the function $f : D \to C$ is onto if and only if for every subset $X \subset C$ we have $f(f^{-1}(X)) = X$.

5. Prove that for all $n \in \mathbb{N}$,
   $$1 + 3 + \cdots + (2n - 1) = n^2.$$

6. Let $x_1, x_2, x_3, \ldots$ be a sequence of numbers defined recursively by
   $$x_1 = 0 \quad \text{and} \quad x_{n+1} = \frac{1 + x_n}{2}.$$ 
   Prove that $x_n < x_{n+1}$ for all $n \in \mathbb{N}$. Can you find a formula for $x_n$?

7. **Bonus problem.** Consider the sequence defined by $a_1 = 1$ and $a_{n+1} = 2a_n + \sqrt{3a_n^2 - 2}$, for any $n \in \mathbb{N}$. Prove that all the terms of the sequence are positive integers.