Modify the problem. Formulate an equivalent problem

Problem 1. A teacher must divide 221 apples evenly among 403 students. What is the minimal number of pieces into which she must cut the apples?

Problem 2. A freight train one mile long goes through a tunnel that is one mile long. If the train is traveling at a speed of 15 miles per hour, how long does it take to pass through the tunnel?

Problem 3. One car is leaving the city A toward the city B and travels with the constant speed 30 miles/hour. Another car is leaving B and travels toward A at the constant speed 20 miles/hour. A bird is leaving A at the same time with the car and travels toward B at the constant speed 40 miles/hour. When meets the car coming from B is changing the direction toward A. When meets the car is changing again the direction, etc. The bird stops whe the two cars meet. Find the distance traveled by the bird knowing the distance between A and B is 100 miles.

Problem 4. (Homework) Every hour, on the hour, a train leaves New York for Philadelphia, while another train leaves Philadelphia for New York. The trip between the two cities takes exactly two hours. When a train goes from New York to Philadelphia, how many of the trains going in the opposite direction will it meet?

Problem 5. Mother is preparing a cake which is rectangle shaped. Father cuts and eats a piece which is rectangle shaped. How can mother cut the cake in two pieces for her two children, with a single straight line cut.

Reformulation: Consider a rectangle ABCD which has a hole of the form of a rectangle MNPQ. Find a line which cuts the region in two pieces with the same area.

Problem 6. Evaluate the sum $2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \ldots + n \cdot 2^n$.

Problem 7. For $a > 1$ let $M_a = \{x > 0 | x^a = a^x \}$. Determine a such that $M_a$ has exactly one element.

Problem 8. Prove that for any positive integer $n$ we have the inequality

$$\sqrt[n]{1 + \frac{\sqrt{n}}{n}} + \sqrt[n]{1 - \frac{\sqrt{n}}{n}} < 2.$$  

Problem 9. If $a$ and $b$ are integers such that $a + \sqrt{b} = \sqrt{15} + \sqrt{216}$ find $a/b$.

Problem 10. Prove that $\frac{\sqrt{20} + 14\sqrt{2}}{2} + \frac{\sqrt{20} - 14\sqrt{2}}{2} = 4$.

Problem 11. Let $P$ be a polynomial and $k$ an odd positive integer. Prove that $P$ is reciprocal if and only if $P^k$ is reciprocal.

Problem 12. Ask a friend to pick a number from 1 to 1000. After asking him 10 questions that can be answered yes or no, you tell him the number. What kind of questions?

Problem 13. Let $(x_n)_n$ be a sequence of real numbers. Prove that

\[ \lim_{n \to \infty} \left( 1 + \frac{1}{x_n} \right)^{x_n} = e \text{ if and only if } \lim_{n \to \infty} x_n (\ln(1 + x_n) - \ln x_n) = 1. \]

Problem 14. Find the minimum value of the set

\[ A = \left\{ a \in \mathbb{R} | \cos x > \frac{1}{x + a}, \forall x \in (0, 1) \right\}. \]