Diophantic Equations

Problem 1. Find all three consecutive integers such that the first one is divisible by 7, the second one by 11, and the third one by 17.

Problem 2. Solve in integers the equation \( y^2 = x^4 + x^3 + x^2 + x + 1 \).

Problem 3. Prove that the equation \( x^2 + y^2 = z^2 + 1 \) has infinitely many integer non-trivial solutions \((x, y, z)\).

Problem 4. For a positive integer \( n > 1 \), show that the numbers 1, 2, \ldots, \( n^2 \) cannot be placed in an \( n \times n \) square so that the product of the numbers in any row or column is the same.

Problem 5. Let \( m \) be a multidigit integer which is equal to the sum of the squares of its digits. How many digits has \( m \)?

Problem 6. Prove that the number 40\ldots9 (with at least one zero) is not a perfect square.

Problem 7. Find with proof all solutions of the equation \( 2^n = a! + b! + c! \) in positive integers \( a, b, c, n \).

Problem 8. Prove that the equation

\[
x^4 + y^4 + z^4 - 2x^2 y^2 - 2x^2 z^2 - 2y^2 z^2 = 24
\]

has no integer solution.

Problem 9. Solve the equation \( 2x^2 + [x] = x^4 \).