Show your work and simplify your answers.

1. (6 pts) Let \( f(x) = x^2 + 2x + 5 \).
   a. Find all critical numbers and the intervals on which the function is increasing and decreasing.

   \[
   f'(x) = 2x + 2 = 2(x+1) = 0
   \]
   if \( x = -1 \).

   So \(-1\) is the only critical number.

   \[
   \text{sign of } f' = -1 + \\
   -1
   \]

   \( f \) is increasing on \([-1, \infty)\)
   and \( f \) is decreasing on \((-\infty, -1]\).

   \[
   \{ \text{1 pt}\}
   \]
   \[
   \{ \text{1 pt}\}
   \]
   \[
   \{ \text{2 pts}\}
   \]

   b. Use the First Derivative Test to find any local maxima and local minima.

   By the first derivative test \( f(-1) = 4 \) is a local minimum.

   \[
   \{ \text{2 pts}\}
   \]

2. (4 pts) Let \( f(x) = x^3 - 3x^2 + 7 \) and observe that \( f'(x) = 3x^2 - 6x \). Identify the intervals on which the graph of \( f \) is concave up and concave down. Find all inflection points.

   Since \( f'(x) = 3x^2 - 6x \), we have

   \[
   f''(x) = 6x - 6 = 6(x-1).
   \]

   We have \( f''(x) = 0 \) if \( x = 1 \) and

   \[
   \text{sign of } f'' = -1 + \\
   1
   \]

   So \( f \) is concave down on \((-\infty, 1)\) and
   concave up on \((1, \infty)\).

   Since the concavity changes at \( x = 1 \), \( (1, f(1)) = (1, 5) \) is an inflection point. (There is also a tangent at \((1, 5)\).)