Please write your name and circle your recitation section time above. Each multiple choice problem is worth 6 points. Put your answer in the blank provided. In the free response problems on the following pages, show all your work clearly and neatly. Blank scratch paper will be provided if needed but may not be submitted.

1. (6 pts) Find the derivative \( \frac{dy}{dx} \) where \( y = \frac{4}{\sqrt{x}} \)
   
   \[
   y = 4x^{-1/2} \quad \Rightarrow \quad \frac{dy}{dx} = 4(-\frac{1}{2})x^{-3/2} = -\frac{2}{x^{3/2}}
   \]

   \( \text{A} \) \( \frac{2}{x^{3/2}} \) \( \text{B} \) \( \frac{2}{x^{3/2}} \) \( \text{C} \) \( 8\sqrt{x} \) \( \text{D} \) \( 4x^{2/3} \) \( \text{E} \) none of these

2. (6 pts) The derivative of the function \( f(x) = \frac{x + 2}{x + 1} \) is
   
   \[
   f'(x) = \frac{(x+1) - (x+2)\cdot 1}{(x+1)^2} = -\frac{1}{(x+1)^2}
   \]

   \( \text{A} \) \( \frac{2x + 3}{(x+1)^2} \) \( \text{B} \) \( \frac{1}{x+1} \) \( \text{C} \) \( \frac{1}{(x+1)^2} \) \( \text{D} \) \( -\frac{1}{(x+1)^2} \) \( \text{E} \) none of these

3. (6 pts) If \( g(t) = \ln(t^6) \), then its derivative \( g'(t) \) is
   
   \[
   g(t) = 6 \ln t \quad \Rightarrow \quad g'(t) = \frac{6}{t}
   \]

   \( \text{A} \) \( \frac{6}{t} \) \( \text{B} \) \( \frac{1}{t^6} \) \( \text{C} \) \( -\frac{6}{t} \) \( \text{D} \) \( 6 \ln(t^6) \) \( \text{E} \) none of these

4. (6 pts) The slope of the tangent line to the graph \( y = x + \frac{3}{x} \) at \( (1, 4) \) is
   
   \[
   y = x + \frac{3}{x} \quad \Rightarrow \quad \frac{dy}{dx} = 1 - \frac{3}{x^2} \quad \Rightarrow \quad m = \left. \frac{dy}{dx} \right|_{x=1} = 1 - \frac{3}{1^2} = -2
   \]

   \( \text{A} \) \( \frac{2}{3} \) \( \text{B} \) \( -\frac{4}{3} \) \( \text{C} \) \( -2 \) \( \text{D} \) 4 \( \text{E} \) none of these
(5) (6 pts) The derivative of the function \( f(x) = \tan^{-1}(3x) \) is
A) \( 3(\sec^{-1} 3x)^2 \)  B) \( \frac{1}{x^2 + 9} \)  C) \( \frac{3}{9 + x^2} \)  D) \( \frac{3}{1 + 9x^2} \)  E) none of these
\[
f'(x) = \frac{1}{(3x)^2} \cdot 3 = \frac{3}{1 + 9x^2}
\]

(6) (6 pts) The derivative of the function \( f(x) = x^4e^x \) is
A) \( 4xe^x \)  B) \( 3x^3(x + 4)e^x \)  C) \( x^4e^x + 4x^3 \)  D) \( 4x^3e^x \)  E) none of these
\[
f'(x) = 4x^3e^x + x^4e^x
= x^3(4 + x)e^x
\]

(7) (6 pts) The derivative of the function \( g(x) = \sin^{-1}(x^3) \) is
A) \( \frac{x^2}{\sqrt{1 - x^6}} \)  B) \( \frac{3x^2}{x^4\sqrt{1 - 1}} \)  C) \( 3x^2 \sin^{-1}(x^3) + \frac{1}{\sqrt{1 - x^6}} \)  D) \( \frac{3x^2}{\sqrt{1 - x^6}} \)  E) none of these
\[
g'(x) = \frac{1}{\sqrt{1 - (x^3)^2}} \cdot 3x^2
= \frac{3x^2}{\sqrt{1 - x^6}}
\]

(8) (6 pts) The derivative of the function \( f(x) = \sin(3 - 2x) \) is
A) \( 2\cos(3 - 2x) \)  B) \( -2\sin(3 - 2x) \)  C) \( -2\cos(3 - 2x) \)  D) \( \cos(-2) \)  E) none of these
\[
f'(x) = \cos(3 - 2x)(-2)
= -2\cos(3 - 2x)
\]
(9) (12 pts) Show that the point $P(1, -1)$ lies on the curve $xy^2 + 3xy + 2 = 0$. Use implicit differentiation to find $\frac{dy}{dx}$ and then find an equation of the line tangent to the curve at $P$.

Check $(x, y) = (1, -1)$

$xy^2 + 3xy + 2 = (1)(-1)^2 + 3(-1) + 2 = 0$ 2pts

Diff implicitly

$y^2 + 2xy \frac{dy}{dx} + 3y + 3x \frac{dy}{dx} = 0$

$(2xy + 3x) \frac{dy}{dx} = -y^2 - 3y$

$\frac{dy}{dx} = -\frac{y^2 + 3y}{2xy + 3x}$

$\left. \frac{dy}{dx} \right|_{(1, -1)} = -\frac{(1)^2 - 3}{2 - 3} = 2$ 2pts

$y = 2(x - 1) - 1$

$y = 2x - 3$ 1pt

(10) (10 pts) Find the rate of change of the area of a square when the edge length is 35 cm and is increasing at 2 cm/s.

$A = x^2 \text{ cm}^2$

When $x = 35 \text{ cm}$, $\frac{dx}{dt} = 2 \text{ cm/s}$

$\frac{dA}{dt} = 2x \frac{dx}{dt} \text{ cm}^2/\text{s}$

$= 2 \cdot 35 \cdot 2 \text{ cm}^2/\text{s}$

$= 140 \text{ cm}^2/\text{s}$

Must have the right units for full credit
(11) (30 pts) Find the derivative of each of the following. Please simplify your answer.

a. \( y = \ln(x^5(3x - 4)^7) \)

\[
y = \ln(x^5(3x - 4)^7) = 5\ln x + 7\ln(3x - 4)
\]

\[
\frac{dy}{dx} = \frac{5}{x} + \frac{7}{3x - 4} \cdot 3
\]

\[
= \frac{5}{x} + \frac{21}{3x - 4}
\]

b. \( f(x) = x \cos 2x \)

\[
\frac{d}{dx} \cos 2x = -\sin(2x) \cdot 2
\]

\[
= -2 \sin(2x)
\]

\[
f'(x) = 1 \cdot \cos 2x + x (-2 \sin 2x)
\]

\[
= \cos 2x - 2x \sin 2x
\]

6 pts

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c. \( h(x) = e^{x^2} \sin x \)

\[
\frac{d}{dx} (e^{x^2}) = 3x^2 e^{x^2}
\]

\[
\frac{d}{dx} (\sin x) = \cos x
\]

\[
h'(x) = 3x^2 e^{x^2} \sin x + e^{x^2} \cos x
\]

\[
= e^{x^2} (3x^2 \sin x + \cos x)
\]

4 pts