Solutions

18 Apr 2018 (100 pts)

TT section time: please circle one
8:30 am, 9:30 am, 3 pm, 4:30 pm

Math 181 — Test IIIA
A Kumjian

Please write your name and circle your recitation section time above. For the multiple choice problems, worth 6 points each, put your answers in the blanks provided (no partial credit). In the free response problems show all your work clearly and neatly. All electronic devices including phones must be off. Blank scratch paper is available but may not be submitted. The last question is extra credit.

1. (6 pts) Suppose that \( f'(x) = 2 \cos x \) and \( f(\pi/2) = 1 \). Then \( f(x) \) is
   A) \(-2 \sin x - 1\)
   B) \(2 \sin x - 1\)
   C) \(-2 \sin x + 1\)
   D) \(2 \sin x + 1\)
   E) none of these

   \[ f(x) = 2 \sin x + C \]
   \[ = f\left(\frac{\pi}{2}\right) = 2 \sin \frac{\pi}{2} + C \]
   \[ = 2 + C \]

   So \( C = -1 \) and \( f(x) = 2 \sin x - 1 \)

2. (6 pts) The absolute maximum for the function \( f(x) = x^2 - 2x + 7 \) on the interval \([2, 5]\) occurs at \( x = \)
   A) 2
   B) 5
   C) 1
   D) 3
   E) none of these

   \[ f'(x) = 2x - 2 = 0 \quad \text{iff} \quad x = 1 \text{ not in } (2, 5) \]
   \[ f(2) = 7 \]
   \[ f(5) = 22 \quad \text{abs max at } 5 \]

3. (6 pts) The function \( f(x) = x^3 - 12x \) is decreasing on the interval
   A) \([-2, 2]\)
   B) \([2, \infty)\)
   C) \((\infty, 2]\)
   D) \((\infty, \infty)\)
   E) none of these

   \[ f'(x) = 3x^2 - 12 = 3(x^2 - 4) = 3(x - 2)(x + 2) = 0 \]
   \[ \text{sign } f' = + - + \]
   \[ \text{decreasing on } [-2, 2] \]

4. (6 pts) The graph of the function \( f(x) = 6x^2 - x^4 \) is concave up on the interval
   A) \((\infty, \infty)\)
   B) \((\sqrt{3}, \infty)\)
   C) \((\infty, -\sqrt{3})\)
   D) \((-1, 1)\)
   E) none of these

   \[ f(x) = 12x - 4x^3 \]
   \[ f''(x) = 12 - 12x^2 = 12(1 - x^2) = 0 \quad \text{iff} \quad x = \pm 1 \]

   \[ \text{sign } f'' = + - + \]
   \[ \text{graph is concave up on } (-1, 1) \]
(5) (6 pts) Determine the limit:

$$\lim_{x \to 0} \frac{x}{e^{2x} - 1}.$$ 

A) 0  B) 2  C) 1/2  D) does not exist  E) none of these

Since $$\lim_{x \to 0} x = 0$$ and $$\lim_{x \to 0} e^{2x} - 1 = 0$$, we apply l'Hopital's rule.

$$\lim_{x \to 0} \frac{1}{2e^{2x}} = \lim_{x \to 0} \frac{1}{2e^{2x}} = \frac{1}{2}.$$ 

(6) (6 pts) Find the absolute minimum value of the function $$f(x) = x + \frac{25}{x}$$ on $$(0, \infty)$$, if it exists.

A) 26  B) 10  C) 2√2  D) none of these  E) none of these

$$f'(x) = 1 - \frac{25}{x^2} = \frac{x^2 - 25}{x^2} = 0 \quad x = \pm 5.$$ 

Only $$x = 5$$ is in $$(0, \infty)$$. We have $$\lim_{x \to 5^-} f(x) = +\infty.$$ 

So $$f(x)$$ has its abs min at $$5$$.

$$f(5) = 5 + \frac{25}{5} = 10.$$ 

(7) (6 pts) Find the general antiderivative of $$f(x) = 6\sqrt{x}$$.

A) $$\frac{3}{\sqrt{x}} + C$$  B) $$3x^{\frac{3}{2}} + C$$  C) $$4x^{\frac{3}{2}} + C$$  D) does not exist  E) none of these

$$F(x) = 6 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = 4x^{\frac{3}{2}} + C.$$ 

(8) (6 pts) Find the maximum value of $$xy$$ given that $$x + y = 12$$.

A) 32  B) 36  C) 40  D) does not exist  E) none of these

$$y = 12 - x \quad \Rightarrow$$

$$g(x) = x(12 - x) = 12x - x^2$$

$$g'(x) = 12 - 2x = 0 \quad \Rightarrow \quad x = 6$$

Note $$y = 6$$ at $$x = 6$$.

$$\text{sign } g'(x) = + -$$

$$\text{abs max } g(6) = 6(12 - 6) = 36.$$
(9) (12 pts) Determine the limit:

\[
\lim_{x \to \infty} \frac{3x + \ln x}{x}.
\]

\[
\lim_{x \to \infty} 3x + \ln x = \infty \quad \lim_{x \to \infty} x = \infty
\]

So we use l'Hopital's rule for the indeterminate form \( \frac{\infty}{\infty} \):

\[
\lim_{x \to \infty} \frac{3x + \ln x}{x} = \lim_{x \to \infty} \frac{3 + \frac{1}{x}}{1} = \lim_{x \to \infty} 3 + \frac{1}{x} = 3
\]

(6 pts)

(3 pts)

(10) (15 pts) Find the absolute maximum and minimum of the function \( f(x) = x^2 - 6x + 7 \) on the interval \([-1, 5]\). Be sure to indicate where these occur.

\[
f'(x) = 2x - 6 = 0 \quad \text{if} \quad x = 3; \quad 3 \text{ is in } (-1, 5).
\]

Checking values at the critical points and the ends of the interval, we have:

\[
f(-1) = 14
\]

\[
f(3) = -2
\]

\[
f(5) = 2
\]

So \( f(-1) = 14 \) is the abs max and \( f(3) = -2 \) is the abs min.

(4 pts)
(11) (25 pts) Let \( f(x) = (x + 1)e^{-x} \). Note that \( f'(x) = -xe^{-x} \) and \( f''(x) = (x-1)e^{-x} \).

a. (5 pts) Find all intercepts.

\[
\begin{align*}
    f(0) &= 0 \quad \text{y-intercept at (0, 0)} \newline
    \text{If } x &= -1 \Rightarrow 0 \quad x \text{-intercept at } (-1, 0)
\end{align*}
\]

b. (5 pts) Find the intervals on which the function is increasing and decreasing.

\[
\begin{align*}
    f(x) &= 0 \quad \text{at } x = 0 \newline
    \text{Sign } f(x) &= -e^{-x} \\
    \text{f incr on } (-\infty, 0] \text{ and decr. on } [0, \infty)
\end{align*}
\]

(3 pts)

c. (5 pts) Find all local extrema (indicate where these occur). Which if any of the local extrema are absolute?

There is a local \underline{max} at 0, \( f(0) = 1 \). It is also an absolute max.

(4 pts)

(1 pt)

d. (5 pts) Find the intervals on which the graph of the function is concave up and concave down.

\[
\begin{align*}
    \text{Sign of } f''(x) &= (x-1)e^{-x} \\
    \text{Graph of } f &\text{ is concave up on } (1, \infty) \text{ and concave down on } (-\infty, 1).
\end{align*}
\]

(3 pts)

e. (5 pts) Find all inflection points (include both coordinates).

We have an inflection pt at \((1, \frac{2}{e})\)

(2 pts)

(1 pt)

(12) (Extra Credit: 5 pts) Sketch the graph of \( f(x) = (x + 1)e^{-x} \). Are there any asymptotes?

\[
\begin{align*}
    \lim_{x \to -\infty} (x+1)e^{-x} &= \lim_{x \to \infty} \frac{x+1}{e^x} = \lim_{x \to \infty} \frac{1}{e^x} = 0 \\
    \text{by } \text{Hôpital's rule} \\
    \text{Horiz. asymptote } y = 0 \\
    \text{Asymptote } y = f(x)
\end{align*}
\]

(2 pts)

(3 pts)