(1) (6 pts) The absolute maximum for the function \( f(x) = x^2 + 2x - 7 \) on the interval \([1, 4]\) occurs at \( x = \) 
A) 1  B) 2  C) 3  D) 4  E) none of these

\[
\begin{align*}
  f(x) &= x^2 + 2x - 7 \\
  f(1) &= 1^2 + 2(1) - 7 = -4 \\
  f(4) &= 4^2 + 2(4) - 7 = 9 \\
  \text{max} &\in (1, 4)
\end{align*}
\]

(2) (6 pts) The function \( f(x) = 3x - x^3 \) is increasing on the interval

A) \([-1, 1]\)  B) \([1, \infty)\)  C) \((\infty, 1]\)  D) \((-\infty, \infty)\)  E) none of these

\[
\begin{align*}
  f'(x) &= 3 - 3x^2 \\
  f'(x) &= 0 \quad x = \pm 1 \\
  f(x) &= \text{inc. on } [-1, 1]
\end{align*}
\]

(3) (6 pts) Suppose that \( f'(x) = 3 \cos x \) and \( f(\pi/2) = 1 \). Then \( f(x) \) is

A) \(-3 \sin x + 1\)  B) \(3 \sin x - 2\)  C) \(-3 \sin x + 4\)  D) \(3 \sin x + 1\)  E) none of these

\[
\begin{align*}
  f(x) &= 3 \sin x + C \\
  1 &= f(\pi/2) = 3 \cdot 1 + C \quad C = -2 \\
  f(x) &= 3 \sin x - 2
\end{align*}
\]

(4) (6 pts) Find the maximum value of \( xy \) given that \( x + y = 14 \).

A) does not exist  B) 48  C) 49  D) 50  E) none of these

\[
\begin{align*}
  y &= 14 - x \\
  g(x) &= x(14 - x) = 14x - x^2 \\
  g'(x) &= 14 - 2x = 0 \quad x = 7 \\
  g(x) &= 7(14 - 7) = 49
\end{align*}
\]
(5) (6 pts) Find the general antiderivative of \( f(x) = 12x^\frac{2}{3} \).

A) \( \frac{2}{3}x + C \) \hspace{1cm} B) \( 8x^\frac{3}{2} + C \) \hspace{1cm} C) \( 6x^\frac{3}{2} + C \) \hspace{1cm} D) does not exist \hspace{1cm} E) none of these

\[
\frac{d}{dx} \left( 12x^\frac{2}{3} \right) = 12x^\frac{1}{3} \\
F(x) = 12x^\frac{3}{2} + C = 8x^\frac{3}{2} + C
\]

(6) (6 pts) The graph of the function \( f(x) = x^4 - 24x^2 \) is concave down on the interval

A) \((-2, 2)\) \hspace{1cm} B) \((\sqrt{3}, \infty)\) \hspace{1cm} C) \((-\infty, -\sqrt{3})\) \hspace{1cm} D) \((-\infty, \infty)\) \hspace{1cm} E) none of these

\[
\begin{align*}
\frac{d}{dx}f(x) &= 4x^3 - 48x \\
\frac{d^2}{dx^2}f(x) &= 12x^2 - 48 \quad \text{critical points:} \quad x = -2, 2 \\
\text{Sign of } \frac{d^2}{dx^2}f(x) &\quad \text{graph is concave down on } (-2, 2)
\end{align*}
\]

(7) (6 pts) Determine the limit:

A) 0 \hspace{1cm} B) 3 \hspace{1cm} C) 1/3 \hspace{1cm} D) does not exist \hspace{1cm} E) none of these

\[
\lim_{x \to 0} \frac{e^{3x} - 1}{x}
\]

Apply l'Hôpital's rule since \( \lim_{x \to 0} x = 0 \) and \( \lim_{x \to 0} e^{3x} - 1 = 0 \):

\[
\lim_{x \to 0} \frac{e^{3x} - 1}{x} = \lim_{x \to 0} \frac{3e^{3x}}{1} = 3
\]

(8) (6 pts) Find the absolute minimum value of the function \( f(x) = x + \frac{16}{x} \) on \((0, \infty)\), if it exists.

A) does not exist \hspace{1cm} B) \( 2\sqrt{2} \) \hspace{1cm} C) 17 \hspace{1cm} D) 8 \hspace{1cm} E) none of these

\[
\begin{align*}
f'(x) &= 1 - \frac{16}{x^2} = \frac{x^2 - 16}{x^2} = 0 \quad \text{critical points:} \quad x = \pm 4 \\
\text{Only } 4 \text{ is in } (0, \infty) \\
\text{From } f'(x) &\quad \text{absolute min:} \quad f(4) = 4 + \frac{16}{4} = 8
\end{align*}
\]
(9) (15 pts) Find the absolute maximum and minimum of the function \( f(x) = x^2 - 6x + 11 \) on the interval \([2, 5]\). Be sure to indicate where these occur.

\[
f'(x) = 2x - 6 = 0 \implies x = 3 \quad \text{which is in } (2, 5).
\]

Checking the CP and end pts:

\[
f(2) = 3 \quad \quad f(3) = 2 \quad \quad f(5) = 6
\]

So abs max is \( f(5) = 6 \) and abs min is \( f(3) = 2 \).

(10) (12 pts) Determine the limit:

\[
\lim_{x \to \infty} \frac{2x + \ln x}{x}
\]

Since \( \lim_{x \to \infty} 2x + \ln x = \infty \) and \( \lim_{x \to \infty} x = \infty \), we apply l'Hôpital's rule in ind. form "\( \frac{\infty}{\infty} \)." So we obtain:

\[
\lim_{x \to \infty} \frac{2x + \ln x}{x} = \lim_{x \to \infty} \frac{2 + \frac{1}{x}}{1} = 2
\]
25

(11) (22 pts) Let \( f(x) = (x - 1)e^x \). Note that \( f'(x) = xe^x \) and \( f''(x) = (x + 1)e^x \).

a. (5 pts) Find all intercepts.

\[ f(0) = -1 \quad \text{so} \quad y\text{-intercept is} \quad (0, -1) \]

\[ (x-1)e^x = 0 \quad \text{if} \quad x = 1 \quad \text{so} \quad x\text{-intercept is} \quad (1, 0) \]

b. (5 pts) Find the intervals on which the function is increasing and decreasing.

\[ \text{sign} \ f'(x) \quad -\quad + \quad \text{note} \ f'(x) = 0 \text{ at } x = 0 \]

\[ f \text{ incr on } [0, \infty) \]

\[ f \text{ decr on } (-\infty, 0] \]

c. (5 pts) Find all local extrema (indicate where these occur). Which if any of the local extrema are absolute?

By 1st deriv. test, \( f'(1) = -1 \) is a loc. min.

It is also the abs. min.

d. (5 pts) Find the intervals on which the graph of the function is concave up and concave down.

\[ \text{sign} \ f''(x) \quad -\quad + \quad \text{at} \quad x = 1 \]

Graph is concave up on \((-1, \infty)\) and concave down on \((-\infty, -1)\).

e. (5 pts) Find all inflection points (include both coordinates).

\[ f(1) = -2e^{-1} \quad \text{so} \quad \text{inflection pt is} \quad (1, -\frac{2}{e}) \]

\[ = -\frac{2}{e} \]

(12) (Extra Credit: 5 pts) Sketch the graph of \( f(x) = (x - 1)e^x \). Are there any asymptotes?

\[ \lim_{x \to -\infty} (x-1)e^x = \lim_{x \to -\infty} \frac{x-1}{e^{-x}} = \lim_{x \to -\infty} \frac{1}{x^{-1}e^{-x}} = 0 \]