1. Find the critical points and the phase portrait of the autonomous first-order differential equation, \( y' = y^2 - 2y - 3 \).

Determine the stability properties of each critical point and sketch typical solution curves in the regions in the \( xy \)-plane determined by the graphs of the equilibrium solutions (be sure to include these in your sketch).

Observe that \( y^2 - 2y - 3 = (y - 3)(y + 1) = 0 \) if and only if \( y = 3 \) or \( y = -1 \). So the critical points are \( 3, -1 \). Since \( y^2 - 2y - 3 > 0 \) for \( x < -1 \) or \( x > 3 \) and \( y^2 - 2y - 3 < 0 \) for \( -1 < x < 3 \), the phase portrait (sketched in class) indicates that \( -1 \) is asymptotically stable while \( 3 \) is unstable. Typical solution curves in the regions in the \( xy \)-plane determined by the graphs of the equilibrium solutions are given below:

2. Find a model for the rate of growth of a population \( P \) as a function of time \( t \) assuming that the birth rate is proportional to the existing population and that there is a constant emigration rate \( E \) (assume zero death rate).

Note that rate of growth of the population is obtained by subtracting the emigration rate from the birth rate. We are given that the birth rate is proportional to the existing population; thus the birth rate must be of the form \( kP \) for some constant \( k \). Hence, our model becomes

\[
\frac{dP}{dt} = kP - E.
\]