Please do the problems in order, write clearly and justify your answers. Please use complete sentences. You may assume that the elementary functions are differentiable.

1. Suppose that \( f \) is differentiable on \((0, \infty)\) and that \( \lim_{x \to \infty} f'(x) = L \) where \( L \in \mathbb{R} \). Prove that 

\[
\lim_{x \to \infty} f(x + 1) - f(x) = L.
\]

(Hint: Use the Mean Value Theorem.)

2. Use l'Hôpital's rule (Theorem 4.4.3) to evaluate the following limit. Check that all hypotheses are satisfied.

\[
\lim_{x \to 0} \frac{\cos x - 1}{x^2}
\]

3. Use regular partitions \( P_n \) and Theorem 5.1.8 to prove that \( f(x) = x \) is integrable on \([1, 3] \) and to evaluate 

\[
\int_1^3 x \, dx.
\]

4. Use the fact that every nondegenerate interval contains both rational and irrational numbers to prove that the function \( g \) given below is not integrable on \([0, 1] \).

\[
g(x) = \begin{cases} 
1 & \text{if } x \in \mathbb{Q} \\
0 & \text{otherwise}
\end{cases}
\]

5. Let \( f : [a, b] \to \mathbb{R} \) be a nonincreasing function (i.e. if \( x \leq y \), then \( f(x) \geq f(y) \)). Prove that \( f \) is integrable. 

(Hint: Show that if \( P_n \) is a regular partition, then \( U(f, P_n) - L(f, P_n) = (f(a) - f(b))(b - a)/n \).)